Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 266 JULY, 1958 No. 1

EARTH-MOON ROCKET TRAJECTORIES *

BY

LOUIS GOLD [1]

ABSTRACT

This imposing three-body problem is attacked in the approximation of an adiabatic limit for which the formalism of securing orbital information is posed. An expression is developed for evaluating flight times for rectilinear orbits, but because of computational difficulty, an approximation based upon an earlier work is called upon. In this manner, lunar flight periods for different launching velocities are established.

1. INTRODUCTION

With the achievement of rocket propulsion bordering on capability for escape flights from the earth, it becomes of immediate practical concern to characterize earth–moon trajectories. In the course of such a study, it was learned that Egorov (1)[2] in fact had been investigating earth–moon orbits. His approach, however, turned out to be quite different from that to be described in this communication; brute force numerical-machine methods were and are being employed by him to delineate the various orbits.

A rigorous treatment of the manifest three body problem is extremely difficult. Even the restricted analysis by Egorov (1) requires laborious machine calculations. The analysis offered in this report may be viewed as a first approximation, being based upon an adiabatic limit which considers the earth and moon as essentially stationary during the rocket flight. This certainly is an upper limit for launching velocities $v_0 \to \infty$. The chief virtue of this approach, apart from the fact that the reduced problem becomes somewhat more tractable, is that it affords a means for delineating those regions of flight conditions for which more detailed numerical and machine calculations may be con-

---

* Presented at the American Rocket Society Semi-Annual Meeting, Los Angeles, Calif., June, 1958.

[1] Senior Scientific Executive, Edgerton, Germeshausen & Grier, Boston, Mass.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

sidered worthwhile. This is to say that the present study lends itself to narrowing down the broad realm of solutions to those of special interest, permitting greater concentration of effort where it may count the most.

For further convenience, aerodynamic drag during the early flight period is neglected with additional simplification introduced by confining attention to impulse launching only. Many other factors are ignored in this first approach such as the earth's departure from sphericity, etc. (2).

In the present paper the emphasis will lie in the direction of setting forth the formalism and deducing some immediately obtainable findings. The more elaborate development will be left to later treatment since the problem, even in its reduced form, is still mathematically quite formidable. Thus specific results in this writing will be largely confined to properties of rectilinear flights such as flight times and landing velocities. In this connection, an upper bound will be drawn in terms of some previous work on escape flights from the earth (2).

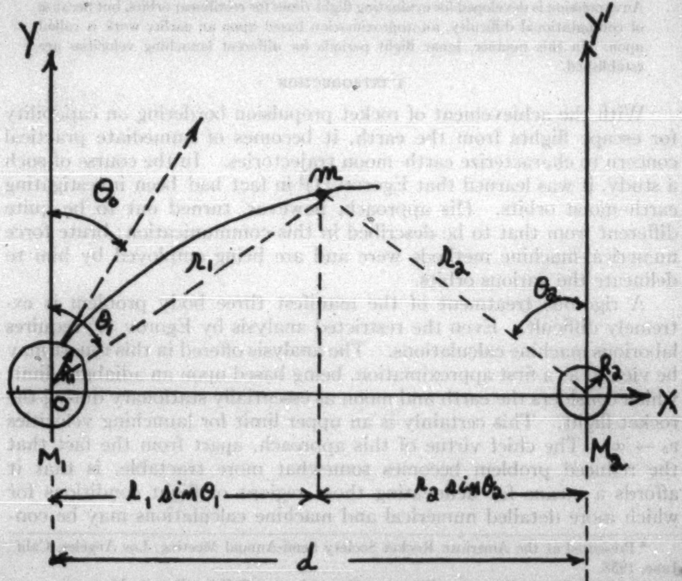


FIG. 1. General spatial configuration for earth–moon flight. M_1 and M_2 denote earth and moon masses with associated radii R_1 and R_2. Rocket position indicated by m; launching angle θ_0.

2. FORMULATION OF THE ADIABATIC METHOD

The physical nature of the problem is contained in Fig. 1. The perturbing influence of the sun is assumed to be negligible. The subscripts 1 and 2 refer to quantities in the earth or lunar system. Thus a hamiltonian in the adiabatic limit may be written in the form

$$H = \tfrac{1}{2}m(\dot{r}_1^2 + r_1^2\dot{\theta}_1^2) - \frac{GM_1m}{r_1} - \frac{GM_2m}{r_2} \tag{1a}$$

$$= \tfrac{1}{2}m(\dot{r}_2^2 + r_2^2\dot{\theta}_2^2) - \frac{GM_1m}{r_1} - \frac{GM_2m}{r_2}. \tag{1b}$$

Since the boundary conditions are

$$t = 0,\ r_1 = R_1;\quad v = v_0 > v_E;\quad \theta = \theta_0 \tag{2}$$

where v_E is the escape velocity, it is sensible to employ the hamiltonian (1a) and eliminate r_2 as follows.

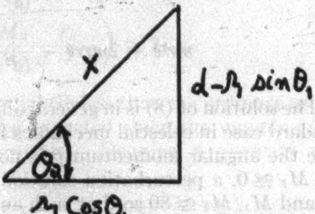


FIG. 2. Defining triangle for Eq. 5.

Referring to Fig. 1, it is seen that

$$\left.\begin{aligned} d &= r_1\sin\theta_1 + r_2\sin\theta_2 \\ 0 &= r_1\cos\theta_1 - r_2\cos\theta_2 \end{aligned}\right\} \tag{3}$$

whence it is found that

$$r_2 = r_1\frac{\cos\theta_1}{\cos\theta_2},\quad \tan\theta_2 = \frac{d - r_1\sin\theta_1}{r_1\cos\theta_1}. \tag{4}$$

Then according to Fig. 2

$$\cos\theta_2 = \frac{r_1\cos\theta_1}{\sqrt{d^2 + r_1^2 - 2dr_1\sin\theta_1}} \tag{5}$$

with the outcome

$$r_2 = (d^2 + r_1^2 - 2dr_1\sin\theta_1)^{1/2}, \tag{6}$$

It is to be noted that the treatment is a two-dimensional one in polar coordinates; if the earth or moon motion were allowed for, a three-dimensional analysis (in spherical coordinates say) would be required. Here is an immediate advantage that accrues in the adiabatic limit.

Dropping the subscripts from r_1 and θ_1, as it will be understood from now on that the earth is the reference frame, the Lagrangian has the appearance

$$\mathcal{L} = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{GM_1m}{r} + \frac{GM_2m}{\sqrt{d^2 + r^2 - 2dr\sin\theta}}. \tag{7}$$

Then the equations of motion stemming from

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}_i}\right) - \frac{\partial\mathcal{L}}{\partial q_i} = 0$$

turn out to be

$$m\ddot{r} - mr\dot{\theta}^2 + \frac{GM_1m}{r^2} + \frac{GM_2m(r - d\sin\theta)}{(d^2 + r^2 - 2dr\sin\theta)^{3/2}} = 0 \tag{8}$$

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} - \frac{GM_2mdr\cos\theta}{(d^2 + r^2 - 2dr\sin\theta)^{3/2}} = 0.$$

The solution of (8) is in general difficult. It obviously reduces to the standard case in celestial mechanics for $M_2 = 0$ and $mr^2\dot{\theta} = \text{const} = p$. Here the angular momentum p is no longer a constant of the motion. For $M_2 \cong 0$, a perturbation solution may suffice; but for the problem at hand $M_1/M_2 \cong 80$ so that such an attack may give limited accuracy in the final results. Elaborate treatment of the general solution is deferred for later consideration.

3. RECTILINEAR EARTH-MOON ROCKET FLIGHTS

Restricting attention to the rectilinear flight defined by

$$t = 0, \quad \theta_0 = \tfrac{1}{2}(2n + 1)\pi = \theta \qquad \text{for all } t \tag{9}$$

the special case for (8) reduces to

$$m\ddot{r} + \frac{GM_1m}{r^2} \mp \frac{GM_2m}{(d \mp r)^2} = 0. \tag{10}$$

Evidently lunar landings correspond to n even with only the minus sign applicable.

Thus a $v(r)$ relation evolves from the formulation

$$\int_{v_0}^{v} v\,dv = G\int_{r_0}^{r}\left(\frac{M_2}{(d - r)^2} - \frac{M_1}{r^2}\right)dr \tag{11}$$

which gives rise to

$$\tfrac{1}{2}(v^2 - v_0^2) = G\left(\frac{M_1}{r} - \frac{M_2}{d - r}\right) + G\left(\frac{M_2}{d - r_0} - \frac{M_1}{r_0}\right). \qquad (12)$$

Now for some value of r a balance between the gravitational pulls of earth and moon will prevail. Taking $v = 0$ at this point means setting $v_0 = v_E$. Hence

$$\frac{M_1}{r} - \frac{M_2}{d - r} = 0, \quad r_E = \frac{M_1}{M_1 + M_2}d \qquad (13)$$

with the outcome

$$v_E^2 = 2G\left(\frac{M_2}{d - r_0} - \frac{M_1}{r_0}\right) \qquad (14)$$

which may be re-written in terms of g_0, the earth's gravitational constant at the surface where $r = r_0$

$$v_E^2 = 2g_0r_0\left(1 - \frac{M_2}{M_1}\cdot\frac{r_0}{d - r_0}\right) \cong 2g_0r_0. \qquad (15)$$

It is to be noted that v_E is actually slightly less than the ordinary escape velocity which of course was to be expected since the moon's gravitational field drags the vehicle onward. Defining a quantity c as $c^2 = v_0^2 - v_E^2$, Eq. 12 takes the form

$$v^2 + c^2 = 2G\left(\frac{M_1}{r} - \frac{M_2}{d - r}\right), \qquad (16)$$

where c^2 is taken always positive for lunar flights.

Next the minimal launching velocity v_m for a rocket to strike the moon obtains when $r = d - R_2$ with $v = 0$ from (12):

$$\tfrac{1}{2}v_m^2 = GM_1\left(\frac{1}{r_0} - \frac{1}{d - R_2}\right) - GM_2\left(\frac{1}{d - r_0} - \frac{1}{R_2}\right). \qquad (17)$$

Then as

$$\frac{1}{r_0} \gg \frac{1}{d - R_2}, \quad \frac{1}{R_2} \gg \frac{1}{d - r_0}$$

a reasonable approximation for (17) becomes

$$v_m^2 \cong 2g_0r_0\left(1 + \frac{g_2R_2}{g_0r_0}\right) \approx 2g_0r_0, \qquad (18)$$

where g_2 is the gravitational constant on the moon's surface. Comparison with v_E in Eq. 15 evidences that

$$v_m^2 \cong v_E^2 + 2g_0 r_0 \frac{M_2}{M_1}\cdot\frac{R_2}{r_0}. \tag{19}$$

4. CHARACTERIZATION OF DIRECT LUNAR FLIGHTS: FLIGHT TIMES AND LANDING VELOCITIES

Equation 16 lends itself to the calculation of lunar flight times for prescribed launching velocities. Thus for rectilinear trajectories

$$\frac{dr}{dt} = \sqrt{2G\left(\frac{M_1}{r} - \frac{M_2}{d-r}\right) - c^2} \tag{20}$$

which yields for $t(r)$

$$t = \frac{1}{c}\int_{r_0}^{r}\sqrt{\frac{r(d-r)}{r^2 - N_1 r + N_2}}\,dr, \quad N_1 = \frac{1}{c^2}(M_1 + M_2) + d \qquad N_2 = 2GM_1 d/c^2 \tag{21}$$

Putting $r = d - R_2$ gives the flight time T.

The evaluation of (21) and the determination of T run into the complication of nasty elliptic integrals which are not easy to handle (see Appendix I). Consequently the rigorous calculation is postponed for later consideration and attention is limited to a lower bound solution which neglects the moon's attraction. For this purpose it can be shown (Appendix II) that escape flights have the following $r(t)$ relations:

1. $v_0 = v^* = \sqrt{2r_0 g_0}$

$$r^{3/2} = r_0^{1/2}(r_0 + \tfrac{3}{2}v^* t) \tag{22a}$$

2. $v_0 > v^*$

$$t = (v_0^2 - v^{*2})^{-1/2}\{\sqrt{r(r+c_0)} - \sqrt{r_0(r_0+c_0)} - c_0 \ln f(r)\} \tag{22b}$$

where

$$c_0 = r_0\left(\frac{v_0^2}{v^{*2}} - 1\right)^{-1}, \quad f(r) = \frac{\sqrt{r + c_0} + \sqrt{r}}{\sqrt{r_0 + c_0} + \sqrt{r_0}}. \tag{22b'}$$

Employing the following data

$$v^* = 7.0 \text{ mi./sec.}, \quad r_0 = 3960 \text{ mi.}, \quad r \cong 240{,}000 \text{ mi.},$$

a rocket dispatched with $v_0 = v^*$ ought to take about

$$T \cong \frac{2}{3}\cdot\frac{1}{7}\cdot\frac{1}{(3960)^{1/2}}\,(24)^{3/2} \times 10^6 \text{ sec.} = 1.8 \times 10^5 \text{ sec.} \cong 50 \text{ hr.}$$

Cole (3) has made such an estimate and gets $t = 49$ hr. The present method can next be utilized for flights where $v_0 > v^*$; a reduced form of (22*b*) is adequate here

$$t \cong \frac{r}{\sqrt{v_0^2 - v^{*2}}}. \tag{22c}$$

This gives rise to further data in Table I.

TABLE I.

T, hr.	50	5.5	1
v_0/v^*	1	2	10

The 50-hr. flight associates as noted to a zero landing velocity. The landing velocities for the more rapid transits may be calculated from (2)

$$\frac{1}{r_0}\left(\frac{v_0^2}{v^{*2}} - 1\right) t = \frac{v}{v^2 - (v_0^2 - v^{*2})} - \frac{v_0}{v^{*2}} - \frac{1}{2\sqrt{v_0^2 - v^{*2}}} \ln Z$$

$$Z = \frac{(\sqrt{v_0^2 - v^{*2}} + v_0)(\sqrt{v_0^2 - v^{*2}} - v)}{(\sqrt{v_0^2 - v^{*2}} - v_0)(\sqrt{v_0^2 - v^{*2}} + v)}. \tag{23}$$

APPENDIX I

To make use of Byrd & Friedman Tables (4) the integral in (21) must be put into the form

$$\int_{r_0}^{r} \sqrt{\frac{(a - t)(t - b)}{(t - c)\ (t - d)}}\, dt, \qquad b = 0. \tag{I.1}$$

Then for all constants real there are four cases

$a > b > c \geqq r > d,$ (4, p. 103)
$a > b > c > r \geqq d,$ (4, p. 107)
$a \geqq r > b > c > d,$ (4, p. 120)
$a > r \geqq b > c > d,$ (4, p. 124).

Additional cases arise when any of constants complex.

A typical illustration has

$$\int_{b}^{v} \sqrt{\frac{(t - c)(t - d)}{(a - t)(t - b)}}\, dt = (b - c)(b - d) g \int_{0}^{u_1} \frac{dn^2 u}{(1 - \alpha^2 sn^2 u)^2}\, du, \tag{I.2a}$$

where

$$\alpha^2 = \frac{a - b}{a - c} < 1, \qquad sn^2 u = \frac{(a - c)(t - b)}{(a - b)(t - c)} \tag{I.2b}$$

$$g = 2(a - c)^{-1/2}(b - d)^{-1/2}.$$

Further evaluation encounters for this special case

$$\int \frac{dn^2u}{(1-\alpha^2 sn^2u)^2}\,du = -\frac{1}{2\alpha^2(\alpha^2-1)}\Big[\alpha^2E(u) + (k^2-\alpha^2)u + (2\alpha^2-\alpha^4-k^2) + \Pi(u,\alpha^2) - \frac{\alpha^4 snucnudnu}{1-\alpha^2sn^2u}\Big] \quad \text{(I.3}a\text{)}$$

with

$$k^2 = \frac{(a-b)(c-d)}{(a-c)(b-d)}. \quad \text{(I.3}b\text{)}$$

Thus it turns out that more effective ways of computing t are to be found. Several schemes for doing so will be dealt with in a later work.

APPENDIX II

A calculation paralleling the one made here may be found elsewhere (**5**); the method given below is more explicit and does not contain peculiar, restrictive boundary conditions. Starting with

$$\tfrac{1}{2}(v^2 - v_0^2) = GM\left(\frac{1}{r} - \frac{1}{r_0}\right) \quad \text{(II.1)}$$

it is easy to find

$$t = \int_{r_0}^{r} \frac{dr}{\sqrt{2GM\left(\frac{1}{r}-\frac{1}{r_0}\right) + v_0^2}}. \quad \text{(II.2)}$$

Three cases must be considered:

1. $v_0 = v^*$ (*critical lunar flight*)

$$t = \frac{1}{v^* r_0^{1/2}} \int_{r_0}^{r} r^{1/2}dr = \frac{2}{3}\cdot\frac{1}{v^* r_0^{1/2}}\,(r^{3/2} - r_0^{3/2}) \quad \text{(II.3)}$$

which yields (22*a*).

2. $v_0 < v^*$ (*earth bound flights*)

$$t = \int_{r_0}^{r} \sqrt{\frac{r}{v^{*2}r_0 - (v^{*2} - v_0^2)r}}\,dr \quad \text{(II.4)}$$

whence

$$t = (v^{*2} - v_0^2)^{-1/2}\left\{\sqrt{r_0(q-r_0)} - \sqrt{r(q-r)} + q\left[\sin^{-1}\sqrt{\frac{q-r}{q}} - \sin^{-1}\sqrt{\frac{q-r_0}{q}}\right]\right\},$$
$$q = r_0\left(1 - \frac{v_0^2}{v^{*2}}\right)^{-1} \quad \text{(II.5)}$$

3. $v_0 > v^*$ (*lunar flights*)

The result (22*b*) follows directly from (I.4) in this situation.

REFERENCES

(1) V. A. EGOROV, *Usp. Fiz. Nauk.*, Vol. 63, p. 73 (1951).
(2) L. GOLD, JOUR. FRANKLIN INST., Vol. 264, p. 301 (1957).
(3) D. M. COLE, *Jet Propulsion*, Vol. 27, p. 176 (1957).
(4) P. F. BYRD AND M. D. FRIEDMAN, "Handbook of Elliptic Integrals for Engineers and Physicists," Berlin, Springer-Verlag, 1954.
(5) W. D. MACMILLAN, "Statics and the Dynamics of a Particle," New York, McGraw-Hill Book Co., 1927, 1st ed., p. 217.

EFFECTS OF UNIPOLAR AIR IONS ON MICROORGANISMS AND ON EVAPORATION *

BY

A. P. KRUEGER,[1] W. WESLEY HICKS,[2] AND J. C. BECKETT [3]

SUMMARY

Bacteria in droplets of distilled water were exposed to unipolar atmospheric ions produced by radioisotopes. Experiments using micro-chemical equipment and bacteriological techniques were carried out both in clean and contaminated air. Lethal results in clean air were attributed primarily to the action of ions on staphylococci. Survival in contaminated air was attributed primarily to the action of contaminants in shielding the staphylococci from (high mobility) ions. Evaporation rates were found to differ according to the presence, or absence, of air ions and atmospheric pollutants.

Although it is well recognized that certain properties of the ambient atmosphere, for example, humidity, greatly influence the fate of airborne microorganisms, the effect of atmospheric electricity has been little more than suspected. At times, conditions would seem to favor the existence of such action for air ions can be present in the atmosphere in considerable concentrations and in various sizes, depending upon prevailing weather conditions, air pollution and ionizing sources. While dried bacteria carried in the air are relatively resistant to most climatic factors (heat, for example), moist cells are considerably more susceptible to both physical and chemical forces. In the present work, micrococci suspended in small drops of distilled water were exposed under controlled conditions to unipolar air ions generated by alpha or beta radiation derived from radioisotopes.

THE BACTERIOLOGICAL EXPERIMENT

During the past two years, Krueger, Smith and Go (1)[4] working in Berkeley, California have conducted bacteriological experiments to detect air ion effects on bacteria. Preliminary work failed to reveal any influence of air ions on such activities as the swarming of *Proteus vulgaris*, the production of hemolysin by streptococci, or the growth rates of various organisms in nutrient media. However, it was found that significant changes in the survival time of staphylococci were produced if the experimental design included the following points:

---

* Presented before the First International Bioclimatological Congress of the International Society of Bioclimatology and Biometeorology, held in Vienna in September, 1957.

[1] Department of Bacteriology, University of California, Berkeley, Calif.

[2] President, Wesix, San Francisco, Calif.

[3] Chief Engineer, Wesix, San Francisco, Calif.

[4] The boldface numbers in parentheses refer to the references appended to this paper.

1. Exposure of cells in droplets sufficiently small to provide a high ratio of surface area to volume.
2. Suspension of washed organisms in distilled water rather than nutrient fluids.
3. Adaptation of microchemical equipment to bacteriological techniques for handling bacteria in small drops and for quantitative estimation of viable cells in fluid aliquots varying from 4 to 100 lambdas (lambda = 10^{-6} liters).

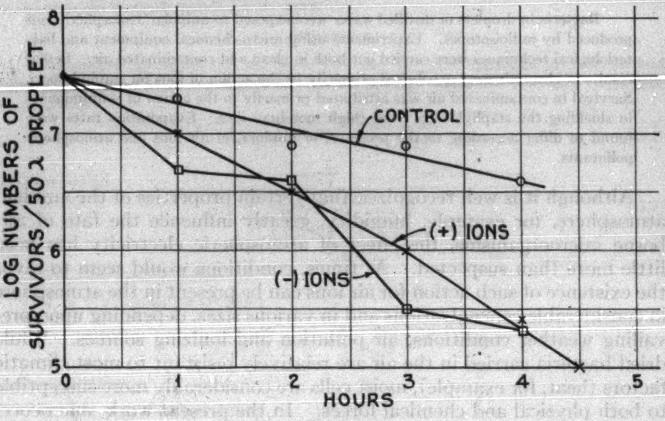


FIG. 1.

Staphylococci were suspended in distilled water and droplets containing 4 to 100 lambdas of the suspension were placed in a glazed ceramic crucible. Ions were produced in the air above the crucible by radioactive means, and the air ions were directed toward the drops in the crucible by an electrostatic field. After exposure to ionized air for various intervals of time, the remaining living organisms were counted by means of replicate plate counts. Results obtained under conditions of clean air were as follows:

1. Positive and negative ions significantly increased the rate of cell death (Fig. 1).
2. No important changes of pH or Eh occurred.
3. The rate of evaporation from the droplets was increased slightly (Fig. 2).

The polonium and tritium sources were used, but by chance the ^{210}Po experiments were done primarily during the period of greatest air pollution (August to November) and the ^{3}H experiments at a time when the air was relatively clean. Differences observed were attributed to the quality of the ambient atmosphere and not the ion source.

In the tritium experiments performed in the absence of air pollutants, the death rate of exposed staphylococci was measurably increased and so was the rate of evaporation. The lethal effect was found to be proportional to the numbers of air ions reaching the surface of the droplet. At 16-cm. distance only 5×10^4 air ions/mm.^{2}/sec. arrive at the surface and this is too small a quantity to induce any measurable biological changes.

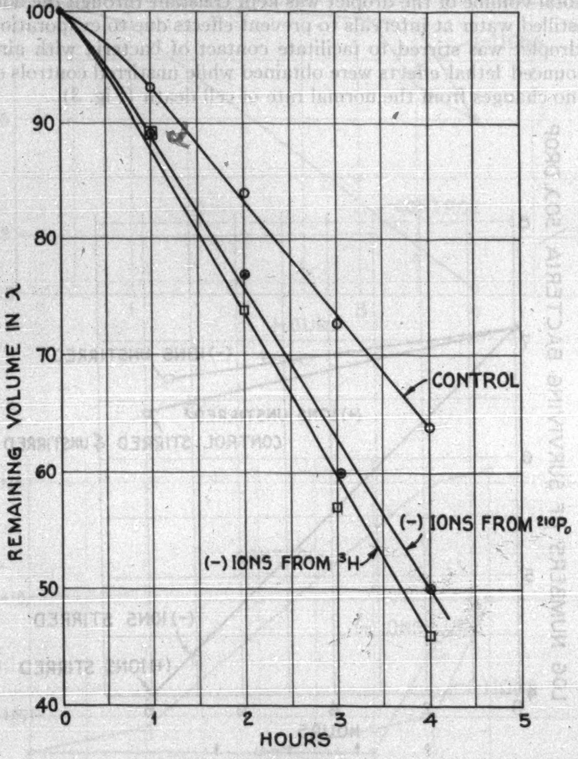


FIG. 2.

The observed increase in rate of evaporation appears to be a reasonable consequence of air ion action, for the motion of air ions in a charged field actually constitutes a minute electrostatic wind and one would anticipate an increase in the kinetic energy of water molecules in the suface layer of the droplets. The tritium experiments further suggest that in a clean atmosphere air ions can act directly on cells. Thus, if the total volume of the droplet was kept constant through the addition of distilled water at intervals to prevent effects due to evaporation and the droplet was stirred to facilitate contact of bacteria with air ions, pronounced lethal effects were obtained while unstirred controls exhibited no changes from the normal rate of cell death (Fig. 3).

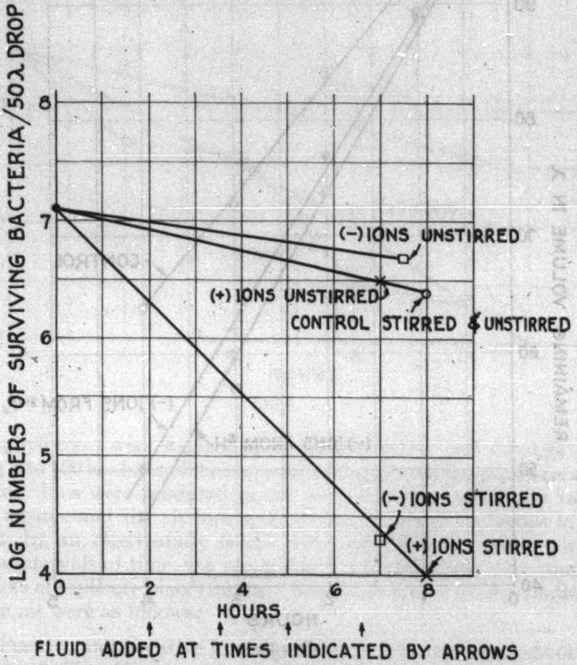


FIG. 3.

The general features of the ^{210}Po series apply to 100 λ droplets of a distilled water suspension of staphylococci in smog-polluted air and may be summarized as follows:

1. Positive and negative ions delayed the fall in pH and accompanying rise in Eh customarily observed in smog-laden air (Figs. 4 and 5).

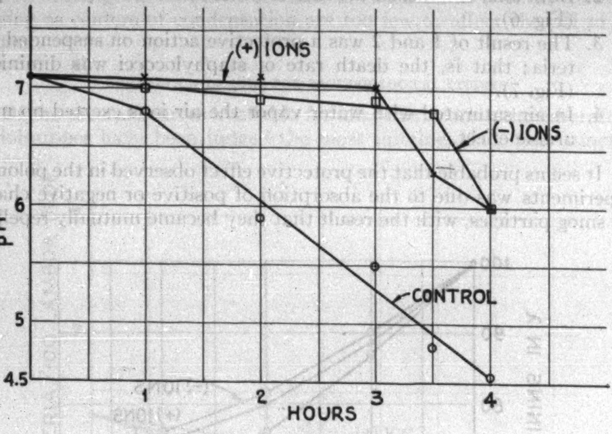

(+) IONS
(-) IONS
CONTROL
pH
7
6
5
4.5
1
2
3
4
HOURS

FIG. 4.

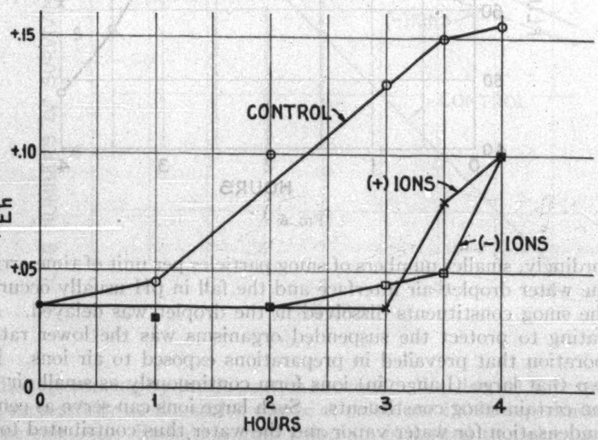

+.15
+.10
+.05
0
Eh
CONTROL
(+) IONS
(-) IONS
0
1
2
3
4
HOURS

FIG. 5.

2. Both ions diminished the rate of water loss from exposed droplets (Fig. 6).
3. The result of 1 and 2 was a protective action on suspended bacteria; that is, the death rate of staphylococci was diminished (Fig. 7).
4. In air saturated with water vapor the air ions exerted no measurable effect.

It seems probable that the protective effect observed in the polonium experiments was due to the absorption of positive or negative charges by smog particles, with the result that they became mutually repellant.

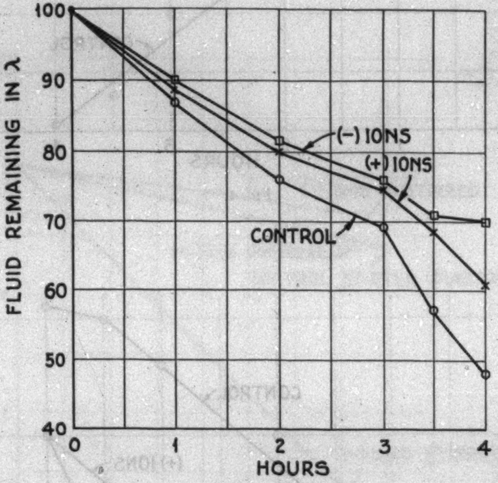


Fig. 6.

Accordingly, smaller numbers of smog particles per unit of time arrived at the water droplet–air interface and the fall in pH usually occurring as the smog constituents dissolved in the droplet was delayed. Also operating to protect the suspended organisms was the lower rate of evaporation that prevailed in preparations exposed to air ions. It is known that large (Langevin) ions form continuously as small air ions act on certain smog constituents. Such large ions can serve as centers of condensation for water vapor and the water thus contributed to the droplet could replace to some degree the fluid lost by evaporation. If this mechanism were to function without inducing a change in pH it must be assumed either that the smog components involved in the formation of Langevin ions are not acidic in character or that the numbers serving as centers of condensation are too few to affect the pH of the droplet.

GENERATION OF IONS FOR BACTERIOLOGICAL STUDIES

Of the several sources available for ion generation in the laboratory, radioisotopes have been judged the most suitable, since they function without producing by-products such as ozone. Worden (2), for example, has successfully used a sealed source of polonium 210 in experiments on tissue cultures. Radiation from polonium is limited to alpha particles travelling a maximum distance of 4 cm. and extremely weak gamma radiation not detectable with ordinary instruments.

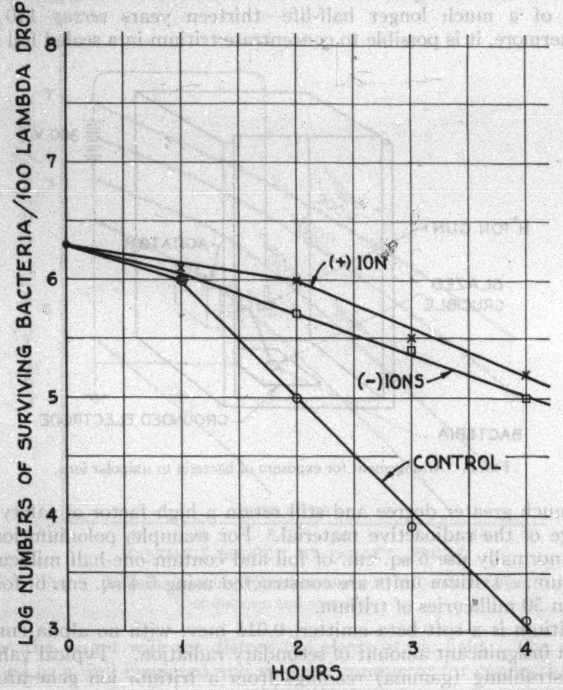


FIG. 7.

Unipolar ions are produced by separating positive and negative ions in an electrostatic field (3). The single-polarity ions are directed to the target area using the same electrostatic field, Fig. 8. By varying the distance from the source material to the target and the potential between source and target, the number of unipolar ions reaching the target can be accurately controlled.

Subsequent to the development of the polonium-type ionizing tube, tritium foil (titanium containing tritium gas) became available and was also used in these experiments. Tritium has the advantage over polonium of a much longer half-life—thirteen years *versus* 140 days. Furthermore, it is possible to concentrate tritium in a sealed foil source to a much greater degree and still retain a high factor of safety from leakage of the radioactive material. For example, polonium ionizing tubes normally use 6 sq. cm. of foil and contain one-half millicurie of polonium. Tritium units are constructed using 0.4 sq. cm. of foil and contain 50 millicuries of tritium.

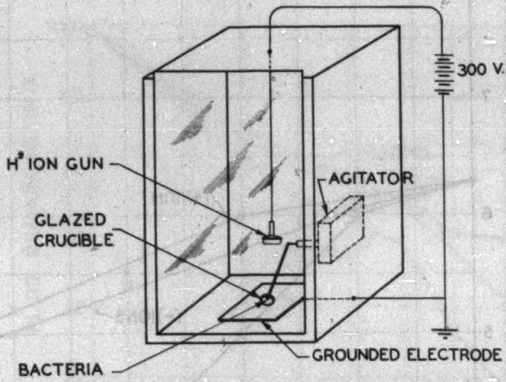


FIG. 8. Arrangement for exposure of bacteria to unipolar ions.

Tritium is a soft beta emitter, 0.015 mev, with no alpha emission and an insignificant amount of secondary radiation. Typical values of Bremsstrahlung (gamma) readings from a tritium ion generator are 5MR/hr. at 6.3 cm., 2.3 MR/hr. at 11.4 cm., and 0.8MR/hr. at 16.5 cm. The small size of the tritium foil itself permits the use of a smaller structure for ion separation and this is of real advantage in studies where space is limited (4). Low energy beta radiation is not as efficient for ionizing air as alpha particles but the greater ease with which tritium can be concentrated in sealed form permits manufacture of more effective tritium ion generators where high density is desired. The relationship of ion density to distance and field potential is shown in Fig. 9.

ION MEASUREMENT

Measurement of atmospheric ions can be made in various ways (5), but for this work a simple target collector was selected as being the most informative. The bacteria are suspended in small droplets in a glazed ceramic crucible and ions are directed toward the crucible as charged particles toward a target.

A metal disk, 25 mm. in diameter, forms the target of the measuring apparatus, Fig. 10. This disk is surrounded by a grounded wire shield so that the target will not distort the electrostatic field. The disk is attached to a well insulated and shielded conductor for connection to a micro-microammeter. Ions reaching the target area are measured in terms of ions/mm.²/sec. as determined from the equation $N = I/qA$ where:

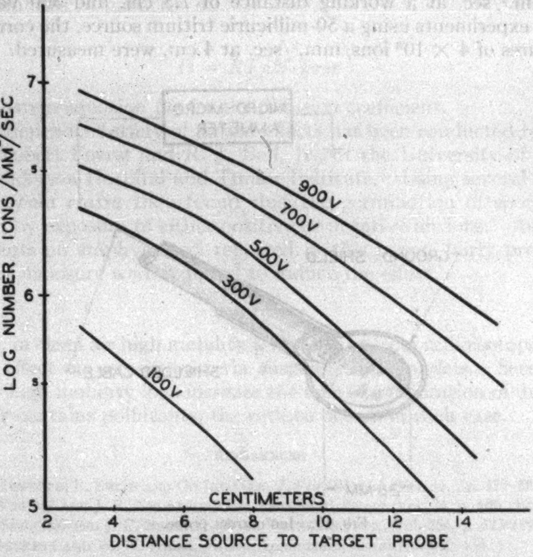


FIG. 9. Unipolar ion density (tritium source 50 MC) as a function of distance and potential.

N = number of ions/mm.²/sec.
I = ion current in amperes
q = charge of one ion—1.6×10^{-19} coulomb
A = area of probe in mm.²

Results using the target collector in an unconfined space are illustrated in the chart, Fig. 9. It is chiefly high mobility ions that reach a target collector, although this device measures the charge of all ions reaching the target, regardless of size.

These experiments were designed to detect possible lethal effects of air ions, and these densities are substantially higher than found in ordinary outdoor air. With about 200 microcuries of polonium foil the air ion density inside the exposure chamber was observed to be 1×10^6 ions/mm.^{2}/sec. at a working distance of 7.5 cm. and 300 volts. In similar experiments using a 50-millicurie tritium source, the corresponding figures of 4×10^6 ions/mm.^{2}/sec. at 4 cm. were measured.

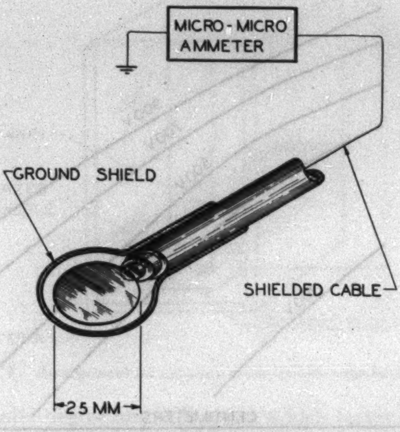


FIG. 10. Ion current probe.

Appropriate steps were taken to avoid two possible sources of error in this type of experimentation: namely, direct action of radiation on the cells, and distortion of viability counts by bacterial agglutination. The distances between the polonium or tritium foils and the test droplets are much greater than the calculated maximal ranges of alpha or beta radiation. Bremsstrahlung or secondary radiation is extremely low from tritium. Likewise, gamma radiation from polonium is barely detectable. As a precaution, control experiments were run with each radioisotope present but without the electrostatic field to effect ion separation. Similarly, experiments were made with the electrostatic field and no radioisotope to insure that neither by itself had any significant effect on the organism. Cell clumping is not observed by microscopic examination and, further, the time periods required to induce lethal effects in light cell suspensions are less than the minimal times for agglutination predicted by the von Smoluchowski equation (6):

$$t = \tfrac{1}{4}\pi D 2 r V_0$$

where

t = time in seconds
r = radius of particle
V_0 = number of particles/ml.

and

$$D = RT/N \cdot \tfrac{1}{6}\pi\eta r$$

is the Einstein equation (7) for the diffusion coefficient.

An independent study of air ion effects has been conducted recently by Dr. Robert Fuerst and R. J. Bell, Jr., at the University of Texas, M. D. Anderson Hospital and Tumor Institute. Using several strains of *Neurospora crassa* they found that the germination of spores was inhibited by exposure to either positive or negative air ions. As in the experiments on staphylococci reported in this paper, fairly prolonged periods of exposure were required to induce the effect.

CONCLUSION

First, in clean air high mobility ions derived from radioisotopes have a lethal effect on certain bacteria suspended in droplets. Second, in clean air high mobility ions increase the rate of evaporation of droplets. If the air contains pollutants, the reverse occurs in each case.

REFERENCES

(1) A. P. KRUEGER, R. SMITH AND GO ING GAN, *J. Gen. Physiol.*, Vol. 41, pp. 359–381 (1957).
(2) J. L. WORDEN AND J. R. THOMPSON, *The Anatomical Record*, Vol. 24, p. 500 (1956).
(3) H. H. SKILLING AND J. C. BECKETT, JOUR. FRANKLIN INST., Vol. 256, p. 423 (1953).
(4) J. C. BECKETT AND W. W. HICKS, Patent Application, Serial No. 640–434.
(5) W. W. HICKS AND J. C. BECKETT, American Institute of Electrical Engineers, *Transactions*, Part I, No. 30, p. 108–111 (1957).
(6) M. VON SMOLUCHOWSKI, *Physik. Z.*, Vol. 17, p. 583 (1916).
(7) A. EINSTEIN, *Ann. Physik*, Vol. 17, p. 549 (1905).

Floating Zone Refining of Thermal Sensitive Intermetallic Compounds.—An improved technique for growing single crystals of binary semiconductors which decompose on melting has been described by J. M. Whelan of Bell Telephone Laboratories. The basic experimental work was performed on gallium arsenide, but the method should be applicable to a variety of compounds which are thermally unstable at their melting points.

According to the paper presented by Dr. Whelan to the 133rd National Meeting of the American Chemical Society in San Francisco, the floating zone method appears to be superior to other methods for growing GaAs single crystals. Composition of the liquid phase at the melting point is strongly dependent on the partial pressure of arsenic. This is most easily controlled by using a sealed system containing excess arsenic and regulating its minimum temperature. The rate of growth is easily controlled and spurious nucleation is greatly reduced by the thermal symmetry of the freezing interface.

Other advantages of the method are the reproducibility, regularity of impurity distributions, and the minimization of apparatus contamination. The relatively large surface-to-volume ratio favors the approach to equilibrium between the liquid and vapor phases. In addition, the small liquid volume minimizes temperature variations in the melt, and the consequent concentration gradients of the principal components.

The technique should be most useful with binary compounds in which only one of the component elements has a considerable vapor pressure at its melting point. The compound must have a high enough electrical conductivity to allow heating by radio frequency induction. The surface tension and density of the molten material must also be such as to support a molten zone during the process.

In the basic floating zone refining technique, a rod is supported vertically. A heat source, for example an induction coil operated at radio frequencies, is moved relative to the rod, melting a liquid zone as it moves. Surface tension supports the liquid zone. Usually by this method, a single crystal can be grown and purification achieved in the zone refining.

Supercharged Hydraulic Oil System Eliminates Surging.—The Yale & Towne Manufacturing Company has announced the perfection of a supercharged hydraulic oil system for lift trucks to eliminate surging in the hydraulic tank and the danger of getting air bubbles into the lines and pump. This new development is being made a standard integrated design feature in all Yale rider-type gas, LP-Gas and electric powered lift trucks.

The presence of air in excessive amounts in a lift truck hydraulic system can lower hoist speeds, cause hoist cavitation and, in extreme cases, lead to pump failure. In the new Yale system, the free flow hydraulic return line brings the hydraulic oil into the tank at a midpoint on the inboard side. A metal pipe conveys the oil straight down into one end of a channel-shaped passage welded to the bottom of the tank. The suction line leading up out of the tank into the hydraulic system is located at the other end of this passage. Action of the hydraulic pump pulls the oil through the passage and into the suction line in a smooth, steady flow. A series of apertures along the sides of the channel passage bleed off excessive oil and pressure smoothly, preventing a bubbling action in the oil remaining in the tank.

V.2
195

PLASTICS IN THE SPACE AGE *

BY

JOHN H. LUX [1]

On October 4, 1957, our radios told us that the Russians had ushered in the Space Age with the first satellite. We all knew it was coming, but we didn't expect it so soon, nor did we expect to find ourselves behind in the "Space Race."

This has required all of us to evaluate our own future contributions so that we can do everything possible to obtain a lead in the near future. We feel that the plastic industry will make major contributions to the "Space Age."

Because of the widespread use of plastics in every phase of our life, many people have suggested we were in the Plastic Age. We have plastics in clothing, automobiles, homes, appliances and food packaging. The word "plastics" brings many different images to peoples' minds, yet to 99 people in a 100, plastics means plates, drinking glasses, toothbrush handles, automobile trim, kitchen work surfaces, toys, or swimming pools. These products have emerged from the laboratories in recent years and are providing us with new and useful items, many of which we never enjoyed before. Plastics in the coming "Space Age" will multiply this list and add many more items and uses, some of which are beyond the scope of our present thinking.

Let's look at the demands of the future. First, there will be a demand for versatile materials of construction for making huge structures which are lighter in weight, unaffected by the elements, easy to handle, transparent or translucent. Second, there will be a demand for materials to operate at temperatures lower than those now generally utilized. Third, there will be a demand for materials to withstand the high temperatures—25,000° F. and perhaps higher. Fourth, there will be a demand for materials which will be expected to stand the high erosion of re-entry and the stress and strains of speeds of high Mach numbers. Fifth, products will be needed for electronic applications with a wide variety of electrical, magnetic and transistor properties. Sixth, there will be a demand for materials which will be able to withstand the nuclear radiation, cosmic rays and other problems encountered in space travel. Seventh, we will need plastics which will provide large amounts of energy through combustion or other chemical reactions for propulsion.

---

* Adapted from a lecture delivered at the Stated Meeting of The Franklin Institute, Wednesday, April 18, 1958.

[1] President, Haveg Industries, Inc., Wilmington, Delaware.

Many of these properties are now available to a wide degree in plastic materials but we look forward to vast improvements in the future.

Before looking at plastics, let's look at why the other materials are unsatisfactory under the above conditions. In the high temperature and high velocity field (areas above 2000° F. and speeds over Mach 1) most people felt that the answer to high temperature materials of construction had the best possibility of coming from such structures as metals, metal carbides, graphite or ceramics. With respect to metals, the thermal conductivity is so high that the metals very quickly reach the temperature at which they lose strength and even melt or flow. Sometimes this happens as quickly as a few parts of a second, particularly above 3000° F., so that above this temperature metals are not practical. In the case of carbides or graphite, these materials at similar high temperatures are subject to oxidation in an oxidizing atmosphere and are also subject to reduction in a reducing atmosphere. Carbides and graphite also have a very high thermal conductivity and both of these materials are subject to serious limitations as far as fabrication techniques are concerned. No large structures can be made without great difficulty and there are limitations as to the strength of even smaller structures. In the case of ceramics, these materials are subject to shock so that in going from room temperatures to 5000° F. in a matter of a second or even less, these materials are subject to cracking.

FIG. 1. Nose cone.

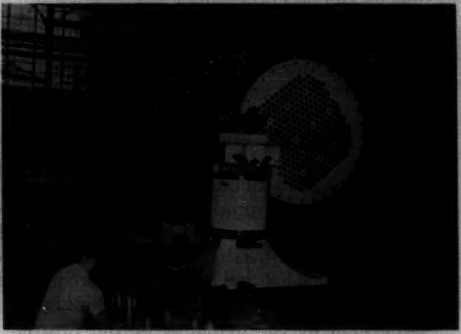


FIG. 2. High temperature heat exchanger system.

In contrast to this, plastic materials and their derivatives have resistance to oxidation and reduction, good thermal insulating qualities, ability to withstand shock, ease of production by a wide variety of techniques, so that they can be fabricated from $\frac{1}{4}$ ounce up to 50,000 pounds and will operate permanently at temperatures up to 550° F., they will operate for thousands of hours in the 600–700° F. range and at lesser times up to 18,000 or 20,000° F. As a result, in the high temperature field, it has been possible to make such materials of construction into blast tubes, re-entry nose cones for missiles, columns, 6000° F. heat insulation, heat exchangers, nozzles, aerodynamic fins, missile jet fins, jet stabilizing rods and many other thermal structural components for aircraft, rockets, missiles and atomic energy processes. (See Figs. 1, 2 and 3.)

In the nuclear field, plastic thermosetting materials have been tested at 10^{10} roentgens for periods of several months and an examination of test samples showed their strength doubled over a period of one year. These are now used in equipment for uranium refining.

In the field of polyethylene resins, it has been found that radiation causes a change in structure which will allow the use of polyethylene to be upgraded from around 130° F. to temperatures above the boiling point of water so they can be sterilized, thus opening up broad new uses.

FIG. 3. Large nozzle.

In the low temperature field, the use of Teflon and Silicone rubber products enables us to produce materials which will operate at −40° and −60° F. and still retain the ability to withstand shock and not be subjected to fatigue as would metals. The silicone polymer oils provide lubrication as low as −60° F.

The reinforced polyester plastics were chosen for the large structures to house the DEW line, early warning radar system for use near the Arctic Circle, because of their ease of assembly, light weight, superior insulating and electrical properties. Many future structures, homes, factories, automobiles, aircraft, and auditoriums will soon burst forth in this versatile combination of plastics and fiberglass.

In the electronic field, Teflon coated wire makes it possible to engineer missile systems which will operate at high temperatures. It also enables computers and other delicate equipment to be built to operate at temperatures which would degrade most insulations.

These things have not been necessary in radios, television and other products because it is easy to call a repair man for a few dollars and replace any defective system. However, as one man has put it: "At 20,000 ft. or 20 miles up there are no repair men." Teflon-coated wire can be welded because in decomposition it does not form carbon particles which would set up conducting points. In the electronic area the use of plastic radomes, electrical terminal insulators, various complex electronic equipment, coaxial cabling and many other items are fabricated from plastics.

The guidance and control system of the missile represents about 50 per cent of its cost (but only 10 per cent of the weight), and it is here that plastics really offer advantages. Plastics are used to make the printed circuits, in eliminating metal parts, for potting and impregnation, and protection of wire and electronic components (see Fig. 4). In the mechanical missile components area there is a wide variety of cams, gears, levers and structural assemblies. In the manufacturing processes, plastics are ideal for tooling, since design changes can be made more quickly and at lower cost than with steel tooling.

FIG. 4. Antenna covers.

Plastic foams, fillers and honeycomb are used in sandwich type construction to provide stronger units. Many metal units, such as the Martin Matador (B-61) are assembled using plastic adhesives. The number of uses of plastic coatings, sealers and liners on other materials will be broadened considerably as new materials are developed.

Also, another feature of the "Space Age" will be more difficult corrosion problems in the manufacture and handling of materials in the future. Plastics, unlike metals, do not corrode, so that they can be made to handle such violent materials as hydrofluoric, hydrochloric and sulfuric acids; strong caustics, chlorine dioxide, aqua regia and chlorine gas. The preparation of uranium, columbium, molybdenum, tungsten and tantalum for "Space Age" use, uses large amounts of plastic equipment.

The development of plastics for the uses of the past and improvements for the future comes about only through the expenditure of a considerable amount of money and technical effort. This does not mean, however, that only large firms can play a significant role in the plastics industry.

My own firm, Haveg Industries, Inc., is a small company, working with plastics since the early 1930's. We have concentrated on solving over one million corrosion problems in the last 25 years, and we have a number of firsts in the industry to our credit: (1) the first plastic pipe (some 25 years ago); (2) the first plastic fittings and valves; (3) the first chemically welded field systems; (4) the first plastic tanks (up to 35,000 gallons); and (5) the world's largest plastic structure, a fume stack weighing 42,000 pounds. This stack, shown in Fig. 5, was field constructed to form a unit 200 feet tall, 5 feet in diameter, designed to withstand a 125-mile wind load. It has been service for several years replacing stainless steel stacks lasting only six months, and at a money saving.

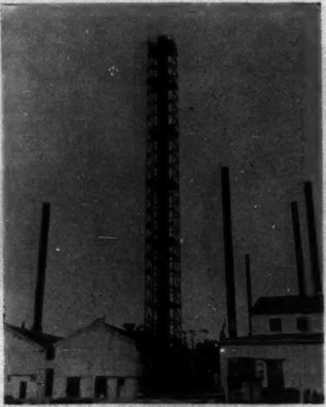


FIG. 5. 200-ft. high temperature stack.

Using over thirty different plastic combinations, we manufacture thousands of items such as tanks, hoods, fume ducts, heat exchangers, cooling towers, reaction vessels, filters, steam jet ejectors, drying trays, electroplating tanks, bifurcators, pumps, rolls, corrosion resistant cements and tank trucks.

It was this background in plastics that caused Haveg to initiate work in the high temperature field many years ago. We felt that plastics were usable at temperatures of 5000° F., for short periods of time—and we proved it in actual rocket-firing tests. Our proprietary materials known as Rocketon, Missileon, Mercuron, Planeton, Galaxon and Asteron for high temperature applications find use in blast tubes, nose cones, nozzles, jet vanes, tail cones, insulators and adaptors (see Fig. 6). In addition to these fabricated materials, we also produce Siltemp glass which is available in both a sheet form and in glass rovings. This material is stable at 1800–2000° F., for use as an ingredient in many military projects and for solving many high temperature problems. Materials can be fabricated for short term use and insulation up to 6000° F. The latter products are usable in areas needing high resistance to erosion.

One of the most outstanding contributions of plastics to our future in the "Space Age" is that of furnishing the energy for our solid propellant type missiles. This field, pioneered by Thiokol Chemical Corporation, has changed rocketry from a "hit and miss" proposition to a field of high reliability. The use of plastics and an oxidizer such as ammonium perchlorate provides good combustion properties, ease of handling, high strength, and ability to work between −40° and +160° F. Among the plastics which are used are polysulfides, polyethylene, epoxy, vinyl, polyurethanes and various synthetic rubbers.

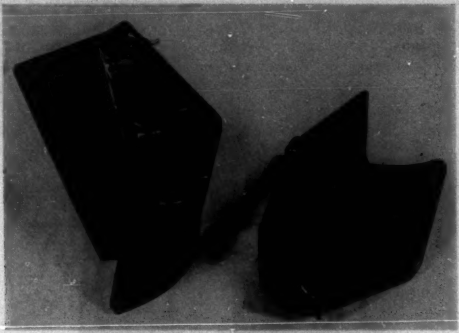


FIG. 6. Jet vanes.

It is obvious that plastics are able to provide the entire missile system with the exception of the wiring and oxidizer; and plastics have been used in the manufacture of both of these.

We have shown you some of the areas of plastics in our future, a future in which even a small organization can play a very significant part and in doing so, can multiply its business from that of a small business to a recognized growth industry in the short period of two to three years.

THE LAGRANGE EQUATIONS IN ELECTRICAL NETWORKS

BY

FREDERICK L. RYDER[1]

SYNOPSIS

The dynamic equations of Lagrange, which are known to apply for certain lumped-element electrical networks, are derived for such networks by a method which results in expressions in which the usual differentiations need not be carried out. These expressions, one suitable for mesh analysis and the other for nodal analysis, are shown to reduce to equations previously obtained by energy considerations in the steady-state case, and, like those equations, are particularly useful when transformers are present. An example is given to illustrate the method of application, as well as to show the relative advantages of the mesh and nodal forms of the expressions.

INTRODUCTION

The familiar Lagrange equations for the analysis of dynamic systems are widely used in mechanical problems, where they yield the following advantages:

1. The derivation of the governing equations of the system is relatively simple because it is formalized once the energy expressions are known. A further, though secondary, aid to simplicity lies in the relative ease of keeping track of polarities, since kinetic and elastic energies are always positive.
2. The solution of the governing equations is relatively compact in that the effects of elements of the mechanical system which absorb no net energy, such as ideal levers, are effectively eliminated from the analysis at an early stage, so that during most of the work the number of parameters is less than in other methods.

It has been shown (1)[2] that the parameters in certain lumped-element electrical networks can be arranged so that Lagrange's equations are satisfied. However, since such networks lack the multidimensionality of mechanical systems, the Lagrange equations offer no advantage over conventional analysis by Kirchhoff's laws—with one exception, namely, when transformers are present. In that case each transformer may be represented by a combination of an ideal transformer and suitable circuit elements by a number of different conventional methods, after which the Lagrange analysis is found easier than the Kirchhoff analysis because the effects of the ideal transformers, which absorb no net energy, may be eliminated early in the analysis.

The electrical form of Lagrange's equations has a further use in connection with certain electrical-structural analogies (2) which are based on the similarity between corresponding electrical and structural expressions for the energy terms in these equations.

---

[1] Member, Scientific Research Staff, Republic Aviation Corporation, Farmingdale, N. Y.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

In the present paper we shall arrive at electrical forms for the Lagrange equations by a method quite different than that used by Jeans, namely by the use of Kirchhoff's laws alone, first by a mesh type of analysis and then by a nodal type. We shall also develop expressions which, while they are equivalent to those of Lagrange, are easier to apply in that the numerous differentiations required in the Lagrange method have already been performed.

MESH ANALYSIS

Let Fig. 1 represent a typical branch *jk* in an electrical network, with constant values of series inductance, resistance and capacitance as

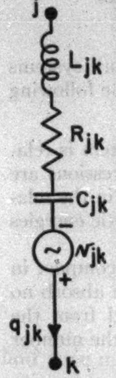


FIG. 1.

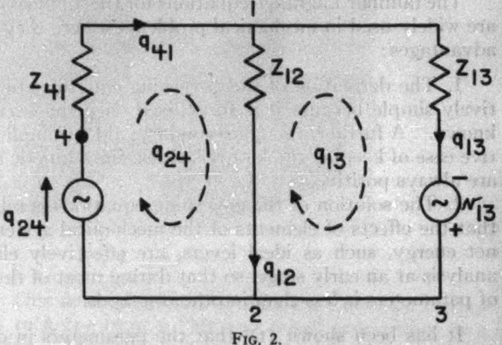


FIG. 2.

indicated. A series voltage difference which rises by the prescribed instantaneous value v_{jk} in the direction j to k is shown, as well as a branch charge of instantaneous value q_{jk}, defined as the time-integral of current flowing in the reference direction. For definiteness we shall stipulate that q_{jk} and v_{jk} denote the respective values of charge and voltage over and above the initial values when time t equals zero. Since charges and voltages are related by linear algebraic or differential equations, the principle of superposition assures that initial and time-dependent values for any charge or voltage may be added to find the total instantaneous value.

Figure 2 shows a simple network containing several branches, with Z denoting the combination of L, R and C for brevity. The generator in branch 1–3 is a source of prescribed voltage of the indicated instantaneous value, the charge q_{13} being unknown. An additional type of element has been added in branch 2–4 in the form of a generator of prescribed charge of instantaneous value q_{24}, the voltage rise being dependent on the remainder of the network. (Such a generator can be simply made up by placing a suitable voltage source in series with a resistance whose voltage drop is large compared with the voltage differences across the other circuit elements.) Note that nodes 2 and 3 have been given separate designations even though they are at the same voltage, so that the various circuit elements may be referred to unambiguously by means of suitable subscripts; this can always be done no matter how complicated the network. Also note that the network is complete within itself; if we wished to enlarge it, say by inserting a current at node 1 and withdrawing it at 2, the complete additional branch would be shown as part of the enlarged network.

By the conventional mesh technique, with mesh *charges* substituted for the usual mesh currents, all branch charges can be evaluated in terms of the prescribed charge plus an additional arbitrarily selected mesh charge. In Fig. 2 the additional mesh charge is shown as q_{13}, which together with q_{24} gives two mesh charges as shown, with the result that

266
58

$$q_{41} = q_{24}$$
$$q_{12} = q_{24} - q_{13}.$$

The unknown mesh charge q_{13} may be considered *redundant* in that it cannot be determined by Kirchhoff's first law (of current-continuity) alone, in the same sense that structural forces and moments are considered redundant when they cannot be determined by equilibrium considerations alone.

These ideas can be extended to more complicated networks, where in general any branch charge q_{jk} can be expressed as follows:

$$q_{jk} = b_{jk1}q_1 + b_{jk2}q_2 + \cdots + b_{jkm}q_m, \tag{1}$$

where m is the number of meshes, and the q's on the right contain all the prescribed charges and as many redundant charges as are necessary. The b's must be either $+1$, -1 or zero, and can be determined either by the conventional mesh technique or by applying Kirchhoff's first law to each node in turn.

The total voltage drop in the reference direction in the typical branch of Fig. 1 is as follows:

$$L_{jk}\ddot{q}_{jk} + R_{jk}\dot{q}_{jk} + q_{jk}/C_{jk} - v_{jk} \equiv Z_{jk}(q_{jk}) - v_{jk} \tag{2}$$

where the dot represents differentiation with respect to time, and Z_{jk} is an appropriate operator. Alternatively, v_{jk} could denote the voltage rise across a charge generator. Summing to zero the corresponding voltage drops, expressed in terms of only prescribed and redundant charges, around the meshes indicated in Fig. 2, we get

$$Z_{13}(q_{13}) - v_{13} - Z_{12}(q_{24} - q_{13}) = 0$$
$$-v_{24} + Z_{41}(q_{24}) + Z_{12}(q_{24} - q_{13}) = 0,$$

where v_{24} is the unknown instantaneous voltage rise across the charge generator in the reference direction. These equations could have been expressed as follows:

$$\sum b_{jks}[Z_{jk}(q_{jk}) - v_{jk}] = 0 \qquad s = 1, 2, \cdots m \tag{3}$$

where the summation is taken *over all branches* for each value of s, which represents each of the m prescribed and redundant currents in turn. For any particular value of s the b's assure that only those branches which form part of the mesh are counted, and also that the direction in which each branch voltage drop is measured is suitably taken into account.

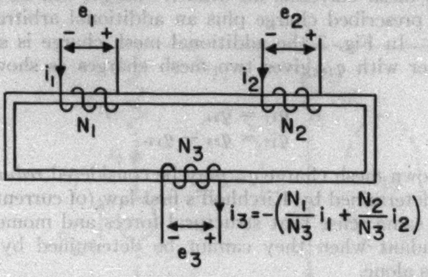

e_1
− +
e_2
− +
i_1
i_2
N_1
N_2
N_3
$i_3 = -\left(\frac{N_1}{N_3} i_1 + \frac{N_2}{N_3} i_2\right)$
− +
e_3

FIG. 3.

Equation 3 expresses the mesh summations of voltage drop in any network, no matter how complicated, made up of the elements shown in Figs. 1 and 2. The resulting m equations are just sufficient to permit evaluation of the redundant charges as well as the unknown voltage rises associated with the charge generators. Since the b's will equal zero when s denotes a redundant charge and jk a branch containing a prescribed charge, the redundants can always be evaluated independently of the voltage rises of the charge generators.

If transformers are present, each of them should first be represented by a combination of an ideal transformer with suitable circuit elements in any one of the numerous conventional ways, after which the current-relationship (which is the same as the charge-relationship if the initial values of charge are ignored) imposed by each ideal transformer on the currents through its various windings should be taken into account in writing Eq. 1. In the particular case of the ideal transformer illustrated

V.2
195

in Fig. 3, where a common flux links three windings having the respective numbers of turns N_1, N_2, N_3 and carrying the respective currents i_1, i_2, i_3, the fact that the net magnetomotive force acting on the core must be zero leads to the following relationship:

$$N_1 i_1 + N_2 i_2 + N_3 i_3 = 0, \tag{4}$$

where the polarity of each of the N's is such that the associated current, when positive, produces flux in a common prescribed direction. The effect of the turns ratios on the current-relationships is shown in Fig. 3, where i_3 is expressed as a function of i_1 and i_2.

Equation 1 cannot now be developed by straightforward application of the mesh symbolism, since in the latter the various transformer winding currents would generally be assumed independent because they would fall in different meshes, whereas they are actually related by expressions such as Eq. 4. However, Eq. 1 can always be developed by considerations of current-continuity at each node. In general this will affect the b's in Eq. 1 so that they may have any constant values, depending on the turns ratios.

To show that Eqs. 3 are valid when ideal transformers are present we shall write it for the case $s = 1$, assuming that the transformer of Fig. 3 is the only one appearing in the network. We get

$$\sum b_{jk1}[Z_{jk}(q_{jk}) - v_{jk}] + 1(-e_1) - \frac{N_1}{N_3}(-e_3) = 0,$$

where the summation term is taken over the entire network except for the transformer windings, and the e's represent transformer voltage rises (measured over and above their values at $t = 0$) as defined in the figure. Since induced voltage is proportional to the number of turns, we may write

$$e_1/N_1 = e_2/N_2 = e_3/N_3 \tag{5}$$

so that the last two terms on the left in the preceding equation sum to zero. This argument can be extended to any configuration of ideal transformers, so that in general the voltage rises of ideal transformers can be omitted in applying Eq. 3. This may yield a considerable saving over conventional methods of analysis, where the e's must be carried along as unknowns even if only the charges (or currents) are to be determined. In the present case the e's are readily calculated, if desired, by voltage-compatibility considerations after the charges have been determined.

In the special case where the prescribed charges and voltage rises are steady-state sinusoids at a given frequency, it is conventional to deal with current vectors **I** rather than the corresponding instantaneous values of charge, and to denote vectorial voltage rises by **V** and complex impedances by **Z**. Then Eqs. 1 and 3 may be replaced by the following:

$$\mathbf{I}_{jk} = b_{jk1}\mathbf{I}_1 + b_{jk2}\mathbf{I}_2 + \cdots + b_{jkm}\mathbf{I}_m \tag{6}$$

$$\sum b_{jks}[\mathbf{I}_{jk}\mathbf{Z}_{jk} - \mathbf{V}_{jk}] = 0 \qquad s = 1, 2, \cdots m. \tag{7}$$

The first of these is identical to Eq. 18 in a previous paper (3) dealing with energy considerations in the steady state; the second takes the place of both Eqs. 16 and 17 in that paper, in that it yields both the redundant currents and the unknown voltage rises of current generators.

We shall now show that Eqs. 3 are equivalent to the well-known Lagrange equations so widely used in solving dynamic mechanical problems. The latter equations can be typified as follows (1, p. 493):

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{p}_s} - \frac{\partial T}{\partial p_s} = \frac{\partial W}{\partial p_s} \qquad s = 1, 2, \cdots m, \tag{8}$$

where T is the kinetic energy of the system expressed as a function of certain selected parameters $p_1, p_2, \cdots p_m$, called generalized coordinates, and their time derivatives (considered for purpose of differentiation as independent parameters). W is the net work done in increasing the kinetic energy of the system, or the difference between the work done by applied forces and that dissipated within the system or stored in it as elastic or potential energy or in any form other than kinetic energy. We shall postulate that inductive energy in the electrical network is analogous to kinetic energy in the mechanical system, thus:

$$T = \tfrac{1}{2}\sum L_{jk}\dot{q}^2_{jk} \tag{9}$$

so that inductance can be considered the analog of mass and current the analog of velocity, from which it can be shown that voltage and force must also be analogous. This corresponds to a well-known and widely used analogy between electrical and mechanical systems.

By Eq. 1 the generalized coordinates in the electrical case are the prescribed and redundant charges $q_1, q_2, \cdots q_m$. Hence the left side of Eq. 8 becomes

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_s} - \frac{\partial T}{\partial q_s} = \sum b_{jks}L_{jk}\ddot{q}_{jk}. \tag{10}$$

The right-hand side of Eq. 8 is called a generalized force, and is conventionally expressed as the increment ΔW of work done (other than kinetic energy) due to an infinitesimal increment Δp, divided by Δp. In the typical electrical branch jk the increment of charge due to Δq_s, say, is $b_{jks}\Delta q_s$ and the voltage rise through which it is forced, with inductance ignored, is $v_{jk} - R_{jk}\dot{q}_{jk} - q_{jk}/C_{jk}$. (Transformer voltage rises may be ignored because, taken together, they are not associated with energy, as can be proved in the particular case of Fig. 3 by summing voltage-current products, taking into account Eqs. 5.) The changes of voltage due to Δq_s are ignored; such changes must take place if q_s is actually varied, but their omission corresponds to the fact that, in the conventional derivation of the Lagrange equations, the increment of the generalized coordinate p_s is considered "virtual" (4) in that it is assumed not to affect the forces within the system. Hence

$$\frac{\partial W}{\partial q_s} = \sum b_{jks}[v_{jk} - R_{jk}\dot{q}_{jk} - q_{jk}/C_{jk}], \tag{11}$$

where the summation extends over all branches as before. By equating the right-hand sides of Eqs. 10 and 11 and comparing the result with Eq. 3, taking account of the definition (2), we see that the Lagrangian equations (8) are correct for the electrical systems under discussion.

NODAL ANALYSIS

Figure 4 shows two typical nodes in an electrical network, with constant values of capacitance, resistance and inductance arranged as indicated. A prescribed current of instantaneous value i_{jk} is maintained by the indicated generator. The time-integral of voltage at the nodes (measured above some convenient datum) will be indicated as f_j and f_k, respectively, where f will for convenience be termed the *flux* (not to be confused with ordinary magnetic flux, although it has the same dimensions). As before, the parameters will be measured over and above the values at $t = 0$.

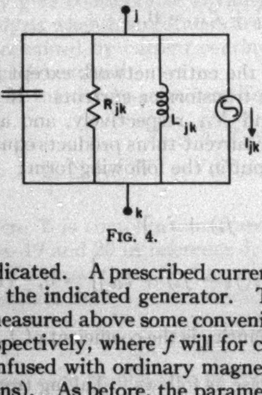


FIG. 4.

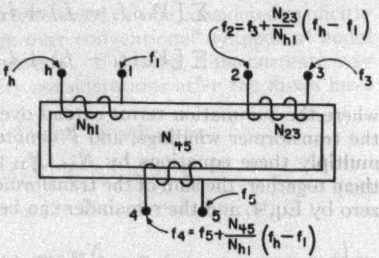

$f_2 = f_3 + \frac{N_{23}}{N_{h1}}(f_h - f_1)$
f_h
h
1
f_1
2
3
f_3
N_{h1}
N_{45}
N_{23}
f_5
4
5
$f_4 = f_5 + \frac{N_{45}}{N_{h1}}(f_h - f_1)$

FIG. 5.

No matter how complicated the network, it is always possible to designate nodes so that each circuit element lies directly between two nodes as in Fig. 4.

The total current flowing in the direction j to k can be shown to equal

$$C_{jk}(\ddot{f}_j - \ddot{f}_k) + (\dot{f}_j - \dot{f}_k)/R_{jk} + (f_j - f_k)/L_{jk} + i_{jk} \equiv Y_{jk}(f_j - f_k) + i_{jk},$$

where Y_{jk} is an appropriate operator. Since the total current leaving node j must be zero, we may write

$$\sum_k [Y_{jk}(f_j - f_k) + i_{jk}] = 0,$$

where k is summed over all nodes other than j. In this expression i_{jk} may also be used to denote unknown current flowing through generators of prescribed flux rise, if any.

If transformers exist, they can be represented by a combination of ideal transformers and appropriate circuit elements as before. Assume that Fig. 5, which is essentially the same as Fig. 3 with designations modified for our present purpose, represents the only transformer within a network. The fluxes at nodes 2 and 4 have been written in terms of those at 3 and 5 and the flux difference between 1 and h by consideration of the turns ratios. Applying the above equation to nodes h, 2 and 4 in turn, we get:

$$\sum_k [Y_{hk}(f_h - f_k) + i_{hk}] + i'_{h1} = 0$$

$$\sum_k [Y_{2k}(f_2 - f_k) + i_{2k}] + i'_{23} = 0$$

$$\sum_k [Y_{4k}(f_4 - f_k) + i_{4k}] + i'_{45} = 0,$$

where the summation terms extend over the entire network except for the transformer windings, and i' denotes transformer currents. If we multiply these equations by N_{h1}, N_{23} and N_{45}, respectively, and add them together, the sum of the transformer current-turns products equals zero by Eq. 4, and the remainder can be put in the following form:

$$\sum_k \left\{ [Y_{hk}(f_h - f_k) + i_{hk}] + \frac{N_{23}}{N_{h1}} [Y_{2k}(f_2 - f_k) + i_{2k}] + \frac{N_{45}}{N_{h1}} [Y_{4k}(f_4 - f_k) + i_{4k}] \right\} = 0, \quad (12)$$

where the coefficient of each bracketed quantity is the coefficient of f_h at the respective nodes in Fig. 5.

We may generalize from this specific case as follows. Taking transformer turns ratios into account, express the flux at any node j thus:

$$f_j = b_{j1}f_1 + b_{j2}f_2 + \cdots + b_{jn}f_n, \quad (13)$$

where the f's on the right contain all prescribed flux differences and as many additional unknown node fluxes (to be termed redundant, like the redundant charges discussed previously) as are necessary to define each node flux. When no transformers exist the b's can equal only zero or plus or minus 1, whereas in the presence of transformers they may have any constant values depending on the turns ratios. In any case the equations corresponding to Eq. 12 may be written as follows:

$$\sum_j \sum_k b_{js}[Y_{jk}(f_j - f_k) + i_{jk}] = 0 \qquad s = 1, 2, \cdots n, \tag{14}$$

where transformer currents may be ignored; the summation with respect to k is repeated for each node j in turn, and s can denote either a redundant node flux or a prescribed flux difference. The resulting equations are just sufficient in number to yield the redundant fluxes and also the unknown currents associated with prescribed flux differences.

To show that the redundants can be evaluated separately from the unknown currents of flux generators, denote each end of such a generator as a node, so that if the flux at one of these nodes is the redundant f_r the flux at the other end is $f_r + f_p$ or $f_r - f_p$, where f_p is the prescribed flux rise. In applying the above equation with $s = r$, the appropriate value of b will be unity at each node, and the current through the flux generator will be counted in a different direction at the two nodes, so that it disappears in the summation.

The fact that transformer currents need not be considered explicitly may give considerable advantage over conventional methods of nodal analysis when transformers are present. If desired, these currents may be obtained by current-continuity considerations after the fluxes have been determined.

In the steady-state case Eqs. 13 and 14 reduce to

$$\left.\begin{aligned} \mathbf{V}_j = b_{j1}\mathbf{V}_1 + b_{j2}\mathbf{V}_2 + \cdots + b_{jn}\mathbf{V}_n \\ \sum_j \sum_k b_{js}[(\mathbf{V}_j - \mathbf{V}_k)\mathbf{Y}_{jk} + \mathbf{I}_{jk}] = 0 \qquad s = 1, 2, \cdots n \end{aligned}\right\}, \tag{15}$$

where $\mathbf{Y}$ is complex admittance. The second of these corresponds to Eqs. 19 and 20 in reference 3.

To show that the Lagrange equations (8) apply to this case, we postulate that

$$T = \tfrac{1}{2} \sum_j \sum_k C_{jk}(\dot{f}_j - \dot{f}_k)^2, \tag{16}$$

where each branch is to be counted only once in the summation. Here capacitance is considered the analog of mass and voltage and analog of velocity, from which it can be shown that current and force must also be analogous. As in the previous case this corresponds to a familiar analogy between electrical and mechanical systems.

By Eq. 13 the generalized coordinates in the electrical case are the prescribed and redundant fluxes. Hence

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{f}_s} - \frac{\partial T}{\partial f_s} = \sum_j \sum_k b_{js} C_{jk}(\ddot{f}_j - \ddot{f}_k). \tag{17}$$

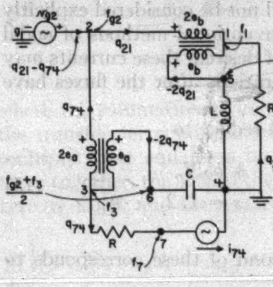
To develop the generalized force corresponding to the right-hand side of Eq. 8, we express ΔW in terms of the incremental flux at each node due to the infinitesimal increment Δf_s, namely, $b_{js}\Delta f_s$, and the total current approaching that node, where the incremental changes in currents are ignored to correspond as before with the procedure used in the conventional derivation of the Lagrange equations. Hence,

$$\frac{\partial W}{\partial f_s} = -\sum_j \sum_k b_{js}[(\dot{f}_j - \dot{f}_k)/R_{jk} + (f_j - f_k)/L_{jk} + i_{jk}], \qquad (18)$$

where transformer currents are ignored because, taken together, they are not associated with energy. By equating the right-hand sides of Eqs. 17 and 18 and comparing the result with Eq. 14, taking account of the symbolism of Y, we see that the Lagrange equations (8) are valid for the electrical network under discussion.

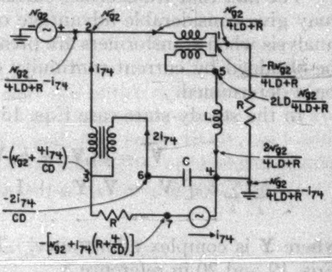
ILLUSTRATIVE EXAMPLES

In Fig. 6 the voltage rise v_{g2} (where g designates ground) and the current i_{74} are prescribed, and the transformers are ideal and their relative numbers of turns are indicated by expressing their voltage rises in terms of e_a and e_b, respectively, the reference direction for the e's being indicated with the aid of a plus sign placed at one end of each winding. The passive circuit elements are as indicated. It is desired to evaluate all branch currents, and all node voltages measured above ground. For simplicity all currents and voltages are considered zero at $t = 0$.



v_{g2}
g
f_{g2}
2
$2e_b$
1
f_1
q_{21}
$q_{21}-q_{74}$
$\frac{f_1+f_{g2}}{2}$
+
e_b
5
$-2q_{21}$
L
R
q_{74}
+
$2e_a$
e_a
$-2q_{74}$
q_{21}
C
$\frac{f_{g2}+f_3}{2}$
3
6
4
f_3
q_{74}
R
7
f_7
i_{74}

FIG. 6.



v_{g2}
2
v_{g2}
$\frac{v_{g2}}{4LD+R}$
1
$\frac{v_{g2}}{4LD+R}-i_{74}$
i_{74}
5
$\frac{-Rv_{g2}}{4LD+R}$
R
$2LD\frac{v_{g2}}{4LD+R}$
L
$2i_{74}$
$-\left(v_{g2}+\frac{4i_{74}}{CD}\right)$
3
6
C
4
$\frac{2v_{g2}}{4LD+R}$
$\frac{v_{g2}}{4LD+R}-i_{74}$
$\frac{-2i_{74}}{CD}$
R
7
$-\left[v_{g2}+i_{74}\left(R+\frac{4}{CD}\right)\right]$
i_{74}

FIG. 7.

Mesh Analysis

The designations of flux should be ignored for the present. By applying Kirchhoff's first law establish that all branch charges can be written in terms of the prescribed charge (which is merely the time-integral of the prescribed current) and one redundant charge. Arbitrarily selecting the latter as q_{21}, and taking due account of the effect of the transformer turns ratios on the charges, we write all branch charges in terms of q_{74} and q_{21} as shown in Fig. 6.

Next applying Eq. 3 with respect to q_{21} and q_{74} in turn, and ignoring transformer voltage rises, we have:

$$\left.\begin{aligned} 1(-v_{g2}) + 1(R\dot{q}_{21}) - 2(-2L\ddot{q}_{21}) &= 0 \\ -1(-v_{g2}) + 1(R\dot{q}_{74} - v_{74}) - 2(-2q_{74}/C) &= 0 \end{aligned}\right\}. \tag{19}$$

As expected, the first of these suffices to determine the single redundant q_{21}, and reduces to:

$$4L\ddot{q}_{21} + R\dot{q}_{21} = v_{g2}$$

so that q_{21} may be evaluated if v_{g2} is specified and the resulting differential equation can be integrated. The branch *currents* are then easily written by Kirchhoff's first law in terms of i_{74} and v_{g2}, with results as shown in Fig. 7, where the symbolic operator $D \equiv d/dt$ is used in conventional algebraic manner.

The second of Eqs. 19 yields the value of v_{74}, which in conjunction with the voltage rises across the inductance, capacitance and resistances, easily calculable now that all charges are known, permits the voltage at all ungrounded nodes to be written, with results as shown in Fig. 7. As a check on the work the reader may evaluate the transformer voltage rises v_{32}, v_{36} and v_{15}, v_{12} and see whether they obey the relationships assumed in Fig. 6.

Nodal Analysis

By consideration of the transformer turns ratios we may express all node fluxes in terms of the prescribed flux f_{g2} (which is merely the time-integral of the prescribed voltage) and the redundant node fluxes, namely, f_1, f_3 and f_7, as shown in Fig. 6, where the charge designations should be ignored for the present purpose. Applying Eq. 14 with respect to the three redundants in turn, and ignoring transformer currents, we have

$$\begin{aligned} 1(\dot{f}_1/R) + \tfrac{1}{2}(f_1 + f_{g2})/2L &= 0 \\ 1(\dot{f}_3 - \dot{f}_7)/R + \tfrac{1}{2}C(\ddot{f}_{g2} + \ddot{f}_3)/2 &= 0 \\ 1(\dot{f}_7 - \dot{f}_3)/R + i_{74} &= 0. \end{aligned}$$

As expected, these equations do not contain the unknown current i_{g2} through the flux generator. The redundants may be expressed as follows:

$$\begin{aligned} 4\dot{f}_1/R + f_1/L &= -f_{g2}/L \\ C\ddot{f}_3 &= -C\ddot{f}_{g2} - 4i_{74} \\ \dot{f}_7/R &= \dot{f}_3/R - i_{74} \end{aligned}$$

so that they can be evaluated in terms of the prescribed current and flux if the differential equations can be integrated. All node *voltages* are then easily written by consideration of the ratios of transformer voltage rises, with results as shown in Fig. 7.

To evaluate i_{g2}, apply Eq. 14 with respect to the prescribed flux difference, obtaining:

$$1(-i_{g2}) + \tfrac{1}{2}(f_1 + f_{g2})/2L + \tfrac{1}{2}C(\ddot{f}_{g2} + \ddot{f}_3)/2 = 0$$

which yields the value shown in Fig. 7. The currents through the inductance, capacitance, and the resistance in branch 41 are found by considering the now-known voltage differences across these elements, and the transformer currents are determined by Kirchhoff's first law. The entire solution checks that previously found by the mesh analysis.

CHOICE BETWEEN MESH AND NODAL ANALYSES

In complicated networks the difficulty of solution is determined largely by the labor of solving the differential equations defining the redundants, since the latter must be evaluated simultaneously. In turn this labor is dependent to a large extent on the order of the matrix of these equations, that is, on the number of redundants. Hence the choice between the mesh and the nodal analyses in any particular case should usually be in favor of that method which yields the fewer redundants.

In the above example there was one redundant in the mesh analysis and three in the nodal analysis, so that the latter was more difficult to apply. The number of redundants in the nodal analysis could have been reduced to two, by considering the combination of the current generator and the resistance in branch 37 as a single new current generator, with an over-all voltage rise v_{34} equal to the sum of the voltage rises in branches 37 and 74; this would have eliminated node 7 during the solution for the redundants. The voltage at 7 could easily have been found by separate calculation after v_{34} was determined.

A corresponding procedure could have been used in the mesh analysis if the voltage generator were shunted by a passive circuit element, by considering the combination of the generator and the additional shunt element as a single new voltage generator, with a single over-all charge equal to $q_{21} - q_{74}$.

REFERENCES

(1) JAMES JEANS, "Electricity and Magnetism," Cambridge University Press, 1927, p. 494.
(2) F. L. RYDER, "Electrical Energy Analogs of Dynamically Loaded Framed Structures," submitted to *J. Aeronautical Sci.*, August, 1957.
(3) F. L. RYDER, "Network Analysis by Least Power Theorems," JOUR. FRANKLIN INST., Vol. 254, p. 47 (1952).
(4) J. L. SYNGE AND B. A. GRIFFITH, "Principles of Mechanics," New York, McGraw-Hill Book Co., Inc., 1942, p. 447.

ON A "BEST" FIVE-POINT DIFFERENCE ANALOGUE OF LAPLACE'S EQUATION

BY

DONALD GREENSPAN[1]

1. INTRODUCTION

A popular technique for the numerical solution of a Dirichlet problem involves the replacement of the Laplace equation

$$u_{xx} + u_{yy} = 0 \tag{1}$$

by the five-point difference analogue

$$-4u_0 + u_1 + u_2 + u_3 + u_4 = 0, \tag{2}$$

where $u_0 = u(x, y)$, $u_1 = u(x + h, y)$, $u_2 = u(x, y + h)$, $u_3 = u(x - h, y)$, $u_4 = u(x, y - h)$, and h is the positive grid constant. (See, for example, reference (**1**),[2] chapters VIII and X, for a complete discussion.) Also, it is known that Eq. 2 is a "best" five-point analogue of Eq. 1, subject to certain assumptions and definitions. (See, for example, (**2**), or the works of Collatz.)

The object here is to generalize the known results and to demonstrate a "best" five-point analogue of Eq. 1 for which the grid size h, in the x direction, need not necessarily be the same as the grid size d, in the y direction.

2. DERIVATION OF THE DIFFERENCE ANALOGUE AND REMARKS ABOUT IT

Let the grid sizes in the x and y directions be h and d, respectively (see Fig. 1). The following notation will be used throughout:

$$\begin{gathered} u_0 = u(x, y), \quad u_1 = u(x + h, y), \quad u_2 = u(x, y + d), \\ u_3 = u(x - h, y), \quad u_4 = u(x, y - d), \end{gathered} \tag{3}$$

$$\frac{\partial^{m+n}u}{\partial x^m \partial y^n} \equiv u_{m,n}; \quad m!n!A_{m,n} = u_{m,n}. \tag{4}$$

Since h and d are assumed to be positive constants, let

$$h = pd \tag{5}$$

where p is a positive constant.

---

[1] Mathematics Department, Purdue University, Lafayette, Ind.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

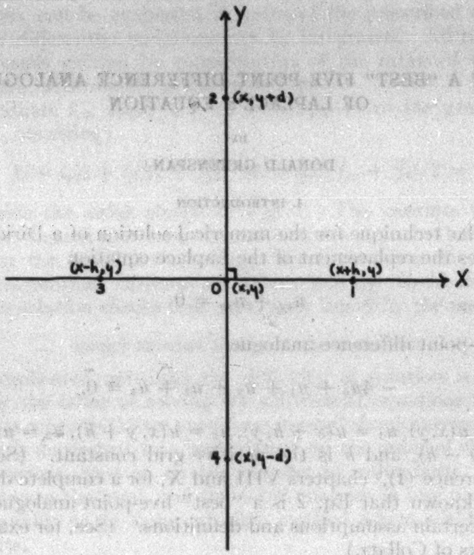


FIG. 1.

Since u is harmonic, then $u_{m+2,n} + u_{m,n+2} = 0$; m, n non-negative integers. Hence:

$$
\begin{array}{llll}
(a) & u_{2,0} + u_{0,2} = 0 & (i) & u_{3,2} + u_{1,4} = 0 \\
(b) & u_{3,0} + u_{1,2} = 0 & (j) & u_{2,3} + u_{0,5} = 0 \\
(c) & u_{2,1} + u_{0,3} = 0 & (k) & u_{6,0} + u_{4,2} = 0 \\
(d) & u_{4,0} + u_{2,2} = 0 & (l) & u_{5,1} + u_{3,3} = 0 \\
(e) & u_{3,1} + u_{1,3} = 0 & (m) & u_{4,2} + u_{2,4} = 0 \\
(f) & u_{2,2} + u_{0,4} = 0 & (n) & u_{3,3} + u_{1,5} = 0 \\
(g) & u_{5,0} + u_{3,2} = 0 & (o) & u_{2,4} + u_{0,6} = 0. \\
(h) & u_{4,1} + u_{2,3} = 0 & &
\end{array}
\qquad (6)
$$

In terms of the A's, (6) may be rewritten, by use of (5), as follows:

$$
\begin{array}{llll}
(a) & A_{2,0} + A_{0,2} = 0 & (i) & 2A_{1,4} + A_{3,2} = 0 \\
(b) & 3A_{3,0} + A_{1,2} = 0 & (j) & 10A_{0,5} + A_{2,3} = 0 \\
(c) & 3A_{0,3} + A_{2,1} = 0 & (k) & 15A_{6,0} + A_{4,2} = 0 \\
(d) & 6A_{4,0} + A_{2,2} = 0 & (l) & 10A_{5,1} + 3A_{3,3} = 0 \\
(e) & A_{3,1} + A_{1,3} = 0 & (m) & A_{4,2} + A_{2,4} = 0 \\
(f) & 6A_{0,4} + A_{2,2} = 0 & (n) & 10A_{1,5} + 3A_{3,3} = 0 \\
(g) & 10A_{5,0} + A_{3,2} = 0 & (o) & 15A_{0,6} + A_{2,4} = 0, \\
(h) & 2A_{4,1} + A_{2,3} = 0 & &
\end{array}
\qquad (7)
$$

or, equivalently,

$$
\begin{array}{llll}
(a) & A_{2,0} = -A_{0,2} & (i) & A_{3,2} = -2A_{1,4} \\
(b) & A_{3,0} = -(1/3)A_{1,2} & (j) & A_{2,3} = -10A_{0,5} \\
(c) & A_{2,1} = -3A_{0,3} & (k) & A_{6,0} = -A_{0,6} \\
(d) & A_{4,0} = A_{0,4} & (l) & A_{5,1} = A_{1,5} \\
(e) & A_{3,1} = -A_{1,3} & (m) & A_{4,2} = 15A_{0,6} \\
(f) & A_{2,2} = -6A_{0,4} & (n) & A_{3,3} = -(10/3)A_{1,5} \\
(g) & A_{5,0} = (1/5)A_{1,4} & (o) & A_{2,4} = -15A_{0,6}. \\
(h) & A_{4,1} = 5A_{0,5} & &
\end{array}
\qquad (8)
$$

Now let

$$L(u) = \sum_{0}^{4} a_i u_i. \qquad (9)$$

For future reference, recall by Taylor series about (x, y), and by use of (5), that:

$$
\begin{aligned}
u_0 &= A_{0,0} \\
u_1 &= A_{0,0} + A_{1,0}pd + A_{2,0}p^2d^2 + A_{3,0}p^3d^3 + A_{4,0}p^4d^4 \\
&\qquad + A_{5,0}p^5d^5 + A_{6,0}p^6d^6 + 0(d^7) \\
u_2 &= A_{0,0} + A_{0,1}d + A_{0,2}d^2 + A_{0,3}d^3 + A_{0,4}d^4 \\
&\qquad + A_{0,5}d^5 + A_{0,6}d^6 + 0(d^7) \qquad (10) \\
u_3 &= A_{0,0} - A_{1,0}pd + A_{2,0}p^2d^2 - A_{3,0}p^3d^3 + A_{4,0}p^4d^4 \\
&\qquad - A_{5,0}p^5d^5 + A_{6,0}p^6d^6 + 0(d^7) \\
u_4 &= A_{0,0} - A_{0,1}d + A_{0,2}d^2 - A_{0,3}d^3 + A_{0,4}d^4 \\
&\qquad - A_{0,5}d^5 + A_{0,6}d^6 + 0(d^7).
\end{aligned}
$$

Substitution of (10) into (9) and recombination yields:

$$
\begin{aligned}
L(u) \equiv {} & A_{00}(a_0 + a_1 + a_2 + a_3 + a_4) + pdA_{1,0}(a_1 - a_3) \\
& + dA_{0,1}(a_2 - a_4) + p^2d^2A_{2,0}(a_1 + a_3) + d^2A_{0,2}(a_2 + a_4) \\
& + p^3d^3A_{3,0}(a_1 - a_3) + d^3A_{0,3}(a_2 - a_4) + p^4d^4A_{4,0}(a_1 + a_3) \\
& + d^4A_{0,4}(a_2 + a_4) + p^5d^5A_{5,0}(a_1 - a_3) + d^5A_{0,5}(a_2 - a_4) \\
& \qquad + p^6d^6A_{6,0}(a_1 + a_3) + d^6A_{0,6}(a_2 + a_4) + 0(d^7). \qquad (11)
\end{aligned}
$$

By use of (8), (11) becomes

$$\begin{aligned} L(u) = {} & A_{0,0}(a_0 + a_1 + a_2 + a_3 + a_4) \\ & + d[A_{1,0}p(a_1 - a_3) + A_{0,1}(a_2 - a_4)] \\ & + d^2 A_{0,2}[(a_2 + a_4) - p^2(a_1 + a_3)] \\ & + d^3[A_{0,3}(a_2 - a_4) - (1/3)p^3 A_{1,2}(a_1 - a_3)] \\ & + d^4 A_{0,4}[p^4(a_1 + a_3) + (a_2 + a_4)] \\ & + d^5[A_{0,5}(a_2 - a_4) + (1/5)p^5 A_{1,4}(a_1 - a_3)] \\ & \qquad + d^6 A_{0,6}[(a_2 + a_4) - p^6(a_1 + a_3)] + 0(d^7). \end{aligned} \tag{12}$$

In order to make (12) of as high an order in d as possible, consider the following set of equations

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 + a_4 = e_1 &= 0(d^4) \\ a_1 - a_3 = e_2 &= 0(d^3) \\ a_2 - a_4 = e_3 &= 0(d^3) \\ a_2 + a_4 - p^2(a_1 + a_3) = e_4 &= 0(d^2). \end{aligned} \tag{12'}$$

The solution of this system in terms of a_0 is:

$$\begin{aligned} a_1 &= \frac{-e_2 - e_2p^2 - e_1 + e_4 + a_0}{-2(1+p^2)}, & a_2 &= \frac{-e_3 - e_3p^2 - e_1p^2 - e_4 + a_0p^2}{-2(1+p^2)}, \\ a_3 &= \frac{a_0 - e_1 + e_2 + p^2e_2 + e_4}{-2(1+p^2)}, & a_4 &= \frac{a_0p^2 - e_1p^2 + e_3 + e_3p^2 - e_4}{-2(1+p^2)}. \end{aligned} \tag{13}$$

Hence, (9) becomes, finally

$$\begin{aligned} L(u) = a_0u_0 & + \left[\frac{-e_2 - e_2p^2 - e_1 + e_4 + a_0}{-2(1+p^2)}\right] u_1 + \left[\frac{-e_3 - e_3p^2 - e_1p^2 - e_4 + a_0p^2}{-2(1+p^2)}\right] u_2 \\ & + \left[\frac{a_0 - e_1 + e_2 + p^2e_2 + e_4}{-2(1+p^2)}\right] u_3 + \left[\frac{a_0p^2 - e_1p^2 + e_3 + e_3p^2 - e_4}{-2(1+p^2)}\right] u_4. \end{aligned} \tag{14}$$

The choice of a_0, e_1, e_2, e_3, e_4 is governed by an attempt to make the right hand side of (11) of as high an order in d as possible. If one chooses $a_0 = -4$, $e_1 = e_2 = e_3 = e_4 = 0$, then $a_1 = a_3 = 2/(1 + p^2)$, $a_2 = a_4 = (2p^2)/(1 + p^2)$. Hence (14) becomes

$$L(u) \equiv -4u_0 + \frac{2}{1+p^2}(u_1 + u_3) + \frac{2p^2}{1+p^2}(u_2 + u_4) \tag{15}$$

while (12) becomes

$$L(u) \equiv 0(d^4). \tag{16}$$

From (15) and (16) then

$$-4u_0 + \frac{2}{1+p^2}(u_1 + u_3) + \frac{2p^2}{1+p^2}(u_2 + u_4) - 0(d^4) \equiv 0. \qquad (17)$$

Discarding the terms $0(d^4)$, one readily deduces from (17) the desired difference equation analogue of (1), that is,

$$-4u_0 + \frac{2}{1+p^2}(u_1 + u_3) + \frac{2p^2}{1+p^2}(u_2 + u_4) = 0. \qquad (18)$$

Note that for the special case $p = 1$, that is, when $h = d$, (18) reduces to (2). It should also be noted that (18) has been derived elsewhere by other methods, but that the techniques used here will prove to be of great value in what follows.

3. ESTABLISHMENT OF EQ. 18 AS A "BEST" ANALOGUE OF LAPLACE'S EQUATION

In the discussion of Section 2, Eq. 17 is the key equation. However, it must, in general, be studied carefully. Suppose, for example, one had set $a_0 = -4d^2$ and $e_1 = e_2 = e_3 = e_4 = 0$ in (14). Then (17) would have taken the form:

$$-4d^2u_0 + \frac{2d^2}{1+p^2}(u_1 + u_3) + \frac{2d^2p^2}{1+p^2}(u_2 + u_4) - 0(d^6) \equiv 0. \qquad (17')$$

The process analogous to that described in going from (17) to (18) would yield in the case of (17′)

$$-4d^2u_0 + \frac{2d^2}{1+p^2}(u_1 + u_3) + \frac{2d^2p^2}{1+p^2}(u_2 + u_4) = 0. \qquad (18')$$

It appears that (18′) results from (17′) by discarding all terms of order at least 6 in d. But, of course, (18′) can be divided by d^2, so that this latter statement is correct, but misleading. Such trouble is avoided as follows. A difference analogue of (1) will be called allowable if and only if

(*a*) it is of the form: $\sum_0^4 a_iu_i = 0$, each u_i defined by (3), and

(*b*) $\lim_{d\to 0} a_i \neq 0$, for at least one value of $i = 0, 1, 2, 3, 4$.

Suppose then that in general one obtains in place of (17):

$$\sum_0^4 a_iu_i + 0(d^n) \equiv 0 \qquad (17'')$$

and in place of (18):

$$\sum_{0}^{4} a_i u_i = 0, \tag{18''}$$

and suppose that (18″) is an allowable difference equation, then (18″) is called an approximation of order $(n-1)$.

It follows immediately that (18) is an allowable third order approximation.

Now it will be shown that (18) is a "best" difference equation analogue of Laplace's equation in that both the following conditions are satisfied:

(*a*) there does not exist an allowable fourth order analogue, and
(*b*) every other third order analogue differs from (18) by only a multiplicative constant.

Theorems 1 and 2, below, establish these results.

Theorem 1. There does not exist an allowable fourth order difference equation analogue for Laplace's equation.

Proof. Suppose such an allowable fourth order approximation exists. It follows from (*a*), (*b*), (*c*), (*d*), (*e*), (*f*), of Eqs. 8, that $A_{0,2}$, $A_{1,2}$, $A_{0,3}$, $A_{1,3}$, $A_{0,4}$ may be viewed as independent parameters for this sub-set of equations, while $A_{2,0}$, $A_{2,1}$, $A_{2,2}$, $A_{3,0}$, $A_{3,1}$, $A_{4,0}$ may be considered dependent variables. Moreover, since $A_{0,0}$, $A_{0,1}$, $A_{1,0}$ are independent of d, it follows that: $A_{0,0}$, $A_{0,1}$, $A_{1,0}$, $A_{0,2}$, $A_{1,2}$, $A_{0,3}$, $A_{1,3}$, $A_{0,4}$ are all independent parameters. Hence, if the right sides of (11), and hence (12), are to be of order at least 5 in d, then it must be that

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 + a_4 &= e_1 = 0(d^5) \\ a_1 - a_3 &= e_2 = 0(d^4) \\ a_2 - a_4 &= e_3 = 0(d^4) \\ (a_2 + a_4) - p^2(a_1 + a_3) &= e_4 = 0(d^3) \\ p^4(a_1 + a_3) + a_2 + a_4 &= e_5 = 0(d^1) = 0(d). \end{aligned}$$

The solution of this sytem is

$$a_0 = e_1 - \frac{e_5 - e_4}{p^2(1+p^2)} - \frac{e_5 + e_4 p^2}{1+p^2}, \quad a_1 = \frac{e_2}{2} + \frac{e_5 - e_4}{2(1+p^2)p^2},$$

$$a_2 = \frac{e_3}{2} + \frac{e_5 + e_4 p^2}{2(1+p^2)}, \quad a_3 = \frac{-e_2}{2} + \frac{e_5 - e_4}{2p^2(1+p^2)}, \quad a_4 = \frac{-e_3}{2} + \frac{e_5 + e_4 p^2}{2(1+p^2)}.$$

Hence, $\lim_{d \to 0} a_i = 0$, for each $i = 0, 1, 2, 3, 4$, by definition of the e_i, $i = 0, 1, 2, 3, 4$. But this is a contradiction to the assumption that an *allowable* fourth order analogue exists. Hence the theorem follows.

Theorem 2. There does not exist an essentially different, allowable third order difference analogue of Laplace's equation other than Eq. 18, that is, every other allowable third order analogue differs from (18) only by a multiplicative constant.

Proof. The discussion of Section 2, through the development of Eq. 14, is that which is sufficient to establish the general third order analogue. Hence consider Eq. 14, rewritten as follows

$$\begin{aligned} L(u) \equiv a_0u_0 - \frac{a_0}{2(1+p^2)}(u_1+u_3) - \frac{a_0p^2}{2(1+p^2)}(u_2+u_4) - \frac{1}{2(1+p^2)} \\ \times[e_1(-u_1-u_2p^2-u_3-u_4p^2)+e_2(-u_1-u_1p^2+u_3+u_3p^2) \\ +e_3(-u_2-u_2p^2+u_4+u_4p^2)+e_4(u_1-u_2+u_3-u_4)]. \quad (19) \end{aligned}$$

By definition of e_1, e_2, e_3, e_4 in (12′), and by use of (10), it follows that

$$\begin{aligned} &e_1(-u_1-u_2p^2-u_3-u_4p^2)=0(d^4) \\ &e_2(-u_1-u_1p^2+u_3+u_3p^2)=(1+p^2)e_2(u_3-u_1)=0(d^3)0(d)=0(d^4) \\ &e_3(-u_2-u_2p^2+u_4+u_4p^2)=(1+p^2)e_3(u_4-u_2)=0(d^3)0(d)=0(d^4) \\ &e_4(u_1-u_2+u_3-u_4)=0(d^2)0(d^2)=0(d^4). \end{aligned} \quad (20)$$

By use of (20), (19) may be rewritten:

$$L(u) \equiv a_0\left[u_0 - \frac{u_1+u_3}{2(1+p^2)} - \frac{p^2(u_2+u_4)}{2(1+p^2)}\right] + 0(h^4). \quad (21)$$

In the same way that (18) was established, it follows that the general, allowable third order analogue is given by

$$a_0\left[u_0 - \frac{u_1+u_3}{2(1+p^2)} - \frac{p^2(u_2+u_4)}{2(1+p^2)}\right] = 0 \quad (22)$$

where a_0 is an arbitrary, non-zero constant. Hence, from Eq. 22, the theorem follows.

REFERENCES

(1) W. E. MILNE, "Numerical Solution of Differential Equations," New York, John Wiley & Sons, Inc., 1953.

(2) D. GREENSPAN, "On a 'Best' Nine-Point Difference Equation Analogue of Laplace's Equation," JOUR. FRANKLIN INST., Vol. 263, pp. 425–430 (1957).

Model Magnified Billion Times.—A model of a molecule magnified one billion times was fabricated by an Air Research and Development Command center for exhibition at the 1958 Brussels World's Fair. It is a model of ferrocene, an American-discovered chemical compound. ARDC's Wright Air Development Center is studying derivatives of ferrocene as fluids and lubricants for space vehicles.

To convert ferrocene into high-boiling liquids, WADC scientists discovered that the compound, which has unusually good resistance to high-temperatures, can be modified to provide lubricant characteristics and a wide liquid range. In early tests of ferrocene compounds as potential lubricants, one derivative of the compound retained its fluidity throughout a 1000 degree temperature range from minus 112 degrees to more than 900 degrees Fahrenheit.

Ferrocene was discovered in 1951 by two chemists, T. J. Kealy and P. L. Pauson of Duquesne University, who came upon it while experimenting with another compound. It was not until several years later when the compound's molecular structure was determined that more extensive research was begun.

Ferrocene, which is an organometallic compound, gets its unusual thermal stability from its molecular arrangement. A "sandwich" compound, considered rare by chemists, ferrocene has proved to be the most stable among the "metal-sandwich" structural types. This unusual structure also makes possible the first known chemical compound composed of carbon, hydrogen and iron.

The name ferrocene is derived from *ferro*, for iron, and *cene*, for benzene which it resembles in chemical reactivity.

Another outstanding property of ferrocene is its anti-soot effect. In tests using ferrocene as an additive to fuel oil, early research workers found no soot or carbon formed in the fire chamber even after prolonged periods of burning.

A ferrocene additive has been investigated for use in jet-engine fuel and oil to eliminate carbon deposits. A compound related to ferrocene is being considered as an anti-knock additive for automobile engines.

WADC's Dr. Harold Rosenberg planned the ferrocene exhibit as a member of the U. S. Molecule (Chemistry) Committee for U. S. participation in the Fair. He is a senior project engineer in the Organic Materials Branch of the Materials Laboratory at WADC.

New Device Simplifies Transistor Mounting.—Transipad, a new device which simplifies installation and permits a more reliable mounting for transistors on printed circuit boards, has been announced by Milton Ross Metals Company, Southampton, Pa.

Designed especially for JETEC 30 transistors, the Transipad is a glass filled Diallyl Phthalate wafer with three holes and three hemispherical feet. The transistor leads pass through the holes to the circuit board solder connections, and the transistor rests directly on the Transipad. This eliminates the necessity of using the leads as supports and permits a lower, more stable mounting with positive insulation between the transistor case and the printed circuit conductors.

The hemispherical feet raise the Transipad off the board, allowing flow clearance for the solder fillets and permitting air to circulate between the circuit board and the Transipad.

THE COSMOLOGICAL PRINCIPLE AND THE COSMOLOGICAL CONSTANT

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

1. INTRODUCTION

Cosmological theories, from Einstein's work in 1917 to the recent models of Bondi and Hoyle, place strong reliance on the so-called "cosmological principle." This is, of course, not a principle of nature; it is a mere hypothesis. The idea, however, is a reasonable one: since we have almost no experimental facts about the cosmos, let us make the simplest general assumption that we can imagine, namely, that the universe appears the same to all observers.

Obviously, this hypothesis is based on little or no direct evidence: indeed, most astronomical observations seem at first sight to deny its validity. The familiar constellations are present only for an observer in the vicinity of Sol. The configuration of the nearby galaxies would be replaced by another configuration, if the observer moved to a different galaxy; and even the clusters of galaxies at the limit of telescopic observation would have a different appearance to a distant observer. If one accepts Mach's principle, this variation in the surroundings may be expected to result in some variation in the laws of nature as measured in different parts of the cosmos. At least, we have no proof that it does not. Thus the cosmological hypothesis, if true at all, has validity only in a statistical sense; and the exact statement of this statistical assumption is not easy.

Various definitions have been formulated in attempts to obtain a rigorous statement. Einstein said (1),[3] "Alle Stellen des Universums sind gleichwertig." Milne obtained greater precision by introducing the idea of a substratum and an associated set of equivalent observers. Two observers are said to be equivalent (2) "when the totality of observations *A* can make on *B* can be described by *A* in the same way as the totality of observations *B* can make on *A* can be described by *B*." Bondi (3) says, "The cosmological principle may now be stated as: Every member of a three-parameter set of fundamental observers obtains the same result of corresponding observations of the universe as every other member."

One may introduce the additional assumption that the cosmos is in statistical equilibrium, so that time as well as position has no effect. Bondi calls this the "perfect cosmological principle." With either the ordinary or the "perfect" hypothesis, the introduction of an idealized substratum lessens vagueness and allows the building up of a deductive science, as has been done by Milne (2). The weakness of this approach seems to lie in its divorce from reality. Everything depends on hypothetical measurements of idealized, non-existent observers; so that one may have an uneasy feeling that the whole deductive system has little relation to the real world.

---

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] Department of Mathematics, University of Connecticut, Storrs, Conn.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

A third criticism of the cosmological hypothesis is that it has not justified the hopes of its originators. Many models of the universe have been devised, but none is satisfactory. To summarize, the cosmological hypothesis may be criticized because

1. It has no direct experimental justification.
2. The difficulty of obtaining a rigorous definition leads to the suspicion that the cosmological hypothesis has little relation to the actual universe.
3. As a tool for the creation of a unique cosmological model, it has been unsuccessful. In fact, as long as we maintain the fundamental requirement that the cosmological hypothesis be satisfied, we are raising an artificial barrier against further progress.

The cosmological hypothesis was introduced as a temporary expedient in view of the dearth of experimental data. But a considerable amount of information has accumulated during the past few years, so that we are now in a much better position than Einstein was in 1917. The time is ripe for an inquiry into the possibility of abandoning the cosmological hypothesis and basing cosmology on experimental facts.

2. THE NEWTONIAN APPROACH

The subject of cosmology is usually associated with the names of Einstein, de Sitter, Eddington, and Lemaître. Their models are based on general relativity and therefore involve a form of the cosmological hypothesis. The original Einstein model was unstable as well as nonexpanding. The expanding model of de Sitter contained no matter and has therefore only the remotest relation to the actual universe. Further modifications by Eddington, Lemaître, Hoyle, and others have failed to produce a unique model that satisfies the requirements.

Certainly, a satisfactory cosmology must not contradict established principles of terrestrial physics, though it may indicate that these terrestrial relations are only special cases of more general laws. Having this idea in mind, we experience a distinct shock when we discover that the expanding relativistic models specify infinite velocities. Experience with particle accelerators shows that the velocity of electrons with respect to the accelerator can never exceed the velocity of light. If the

V.2
195

velocity of elementary particles has an upper limit it is likely that the velocity of galaxies is likewise limited.

For this and other reasons, we tentatively reject cosmologies based on general relativity. The physicist and the engineer employ Newtonian mechanics in dealing with all ordinary macroscopic phenomena; and it is only reasonable to employ these familiar relations as the basis for a large-scale, cosmological formulation, as has been done by Milne and McCrea (4). In taking this step from the moderately large to the very large, we may expect that some modification will be necessary in Newton's equations; but the more general equations should reduce to Newton's for moderate distances. Indeed, one of the principal fruits of cosmological study may well be the additional light it can throw on the equations of every-day physics.

We propose to modify Newton's gravitational equation by the inclusion of a factor $\Phi(\xi)$:

$$\mathfrak{F}_2 = \mathbf{a}_\xi G \frac{m_1 m_2}{\xi^2} \Phi(\xi), \tag{1}$$

where

$\mathfrak{F}_2$ = force on a concentrated mass m_2, caused by mass m_1,
$\mathbf{a}_\xi$ = unit vector pointing from m_1 to m_2,
ξ = distance between m_1 and m_2,
$\Phi(\xi)$ = a gravitational function having the value -1 for small distances and becoming positive for galactic distances.

Evidently some such modification must be made in cosmology to account for the recession of the galaxies.

Equation 1 applies to two mass-points. For a continuous distribution of matter of density ρ, the force on m_2 becomes

$$\mathfrak{F}_2 = m_2 G \int \frac{\mathbf{a}_\xi \rho \Phi(\xi)}{\xi^2} d\mathfrak{V}. \tag{2}$$

For the special case of a symmetric, spherical universe with $\rho = \rho(t)$ and Φ = const,

$$\mathfrak{F}_2 = m_2 \rho G \Phi \int \frac{d\mathfrak{V}}{\xi^2}$$

$$= \frac{4\pi}{3} m_2 \rho G \Phi r, \tag{2a}$$

where r is the distance from m_2 to the center O of the cosmos. Equation 2a, obtained by direct integration, agrees with the usual theory of gravitational potential. The shell of matter outside r has no effect on

m_2, and the force is as if the total mass of a sphere of radius r were concentrated at O. This simple result, however, is obtained only if the density is uniform and $\Phi = \text{const.}$

In the Newtonian formulation of cosmology, we use the equation of motion and the equation for the conservation of matter:

$$\begin{cases} \mathfrak{F}_2 = m_2 \dfrac{d\mathbf{v}}{dt} = m_2 G \displaystyle\int \dfrac{\mathbf{a}_\xi \rho \Phi(\xi)}{\xi^2}\, d\mathfrak{V}, & (3) \\[2ex] \operatorname{div}(\rho \mathbf{v}) = -\dfrac{\partial \rho}{\partial t}. & (4) \end{cases}$$

It is customary to employ a spherically symmetric model of finite radius in euclidean 3-space. Velocity is radial and ρ is a function of time only. Then the foregoing equations become

$$\begin{cases} v \dfrac{\partial v}{\partial r} + \dfrac{\partial v}{\partial t} = G\rho(t) \displaystyle\int \dfrac{\Phi(\xi)}{\xi^2}\, d\mathfrak{V}, & (3a) \\[2ex] \dfrac{1}{r^2} \dfrac{\partial}{\partial r}(r^2 v) = -\dfrac{1}{\rho} \dfrac{d\rho}{dt}. & (4a) \end{cases}$$

Solution of Eqs. 3*a* and 4*a* gives the density $\rho(t)$ and the velocity $v(r, t)$.

It is customary to assume that the universe is expanding, with velocity directly proportional to radius:

$$v = F(t) \cdot r. \tag{5}$$

Substitution into Eqs. 3*a* and 4*a* yields

$$\begin{cases} F^2 + F' = \dfrac{G\rho}{r} \displaystyle\int \dfrac{\Phi(\xi)}{\xi^2}\, d\mathfrak{V}, \\[2ex] F = -\dfrac{1}{3\rho} \dfrac{d\rho}{dt}. \end{cases}$$

If $\Phi = \text{const}$, elimination of ρ results in the differential equation,

$$\frac{d^2F}{dt^2} + 5F \frac{dF}{dt} + 3F^3 = 0. \tag{6}$$

Note that Eq. 6 is independent of the value of $\Phi = \text{const}$. Thus if one assumes linearity, Eq. 5, the velocity distribution is given directly by a solution of Eq. 6, irrespective of Φ.

3. EXPANDING MODEL, MILNE I

As a simple example, consider a modification of the model developed by Milne (4) in 1934. The universe consists initially of tightly packed material in a sphere of radius r_0; and this sphere explodes at $t = 0$, sending out fragments having all velocities from 0 to c.[4] The center of the universe is taken as the origin O of a spherical coordinate system.

Assume that the gravitational force is expressed by the usual Newtonian formula. Then $\Phi = -1$ and the right side of Eq. 3*a* becomes

$$-\frac{4\pi}{3} G\rho(t)r,$$

in accordance with Eq. 2*a*. A solution of the problem, obtained from Eqs. 6, 3*a*, and 4*a*, is

$$\begin{cases} v = \dfrac{2}{3}\dfrac{r}{t+a}, & (7) \\[2ex] \rho = \dfrac{1}{6\pi G(t+a)^2}, & (8) \end{cases}$$

where a is an arbitrary constant.

At any instant t, the total mass M of the universe is contained in a sphere of radius $r_1(t)$. Thus

$$M = \frac{4\pi}{3}\rho(t)r_1^3(t)$$

or

$$r_1(t) = \left[\frac{9}{2} MG\right]^{1/3} (t+a)^{2/3}. \tag{9}$$

The maximum velocity, at a given t, occurs at $r = r_1(t)$; and from Eqs. 7 and 9,

$$v_{\max} = \frac{2}{3}\frac{r_1(t)}{t+a} = \left[\frac{4}{3} MG\right]^{1/3} (t+a)^{-1/3}. \tag{10}$$

The initial condition is now introduced. At $t = 0$, $v_{\max} = c$. Thus, from Eqs. 10 and 9,

$$a = \frac{4MG}{3c^3}, \tag{11}$$

$$r_0 = \frac{2MG}{c^2}. \tag{12}$$

---

[4] Milne assumed that velocities ranged from 0 to ∞ and that $r_0 = 0$.

Each galaxy is undergoing continuous deceleration, and

$$\frac{dv}{dt} = -\frac{2}{9}\,\frac{r}{(t+a)^2}. \tag{13}$$

The behavior of the model is indicated in Figs. 1 to 3.

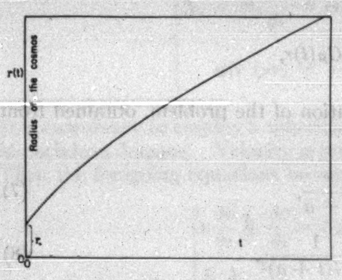


FIG. 1. Radius of the cosmos, according to the expanding model, Milne I.

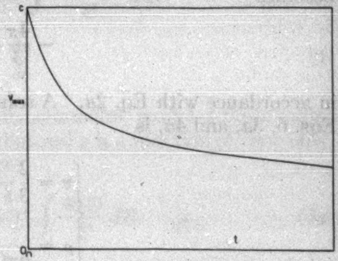


FIG. 2. Maximum velocity of galaxies (Milne I).

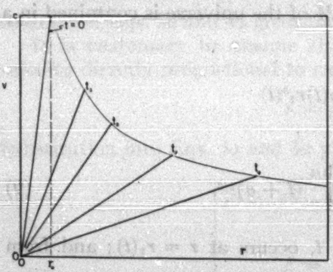


FIG. 3. The velocity distribution is a linear function of radius, but the slope of the line decreases with time (Milne I).

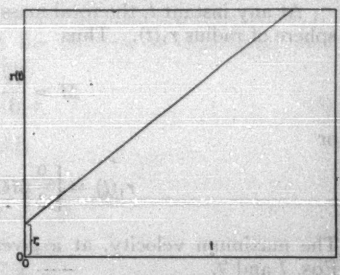


FIG. 4. Radius of the cosmos according to the unaccelerated expanding model (Milne II).

4. FINITE OR INFINITE?

Note that the simple model of Section 3 is obtained without any modification in the ordinary Newtonian equations. Also, it satisfies Olbers' principle, galactic velocities are directly proportional to distances, and maximum velocity does not exceed *c*.

But, strangely enough, the model fails to show the one thing that Milne was trying to prove, namely, that the models of general relativity can be obtained exactly by Newtonian methods. He says (4), "In this paper I show how the same locally observable results can be obtained from elementary Newtonian theory as are given by Einstein and de

Sitter's well-known relativistic model. . . ." Also, ". . . these [equations] are identical with the relativistic equations of an expanding universe of zero curvature with pressure $p = 0$ and cosmical constant $\lambda = 0$, as given by Einstein and de Sitter." This conclusion has been generally accepted. Actually, Milne I is not the Einstein–de Sitter universe, nor does it satisfy the cosmological hypothesis.

A basic characteristic of relativistic cosmologies is that every point is equivalent to every other point. But the Milne models have a *preferred point*, the origin O from which the expansion takes place, and this point is uniquely distinct from all other points. One can easily prove (5) that, for any system satisfying the linear relation, Eq. 5, each observer will see the other galaxies moving away from him on all sides and will observe a statistical isotropy. But this is true only if the limit of his telescope is less than the apparent distance to the boundary of the universe. And the Milne universes do have a sharply defined boundary, beyond which there is nothing. An observer near the edge of the universe will certainly not obtain the same picture as an observer at O. In other words, the cosmological hypothesis is not valid, except in a very restricted sense.

A similar criticism of the Milne derivation was proposed by Layzer (6). In reply, McCrea (5) claimed that though the derivation was made for a finite sphere, the radius r_1 can be allowed to expand to infinity; in which case, the cosmological hypothesis holds. But if $r_1 = \infty$, symmetry and homogeneity require that the gravitational force be zero at any point. Then the right side of Eq. 3*a* becomes zero, and Eqs. 7 and 8 are no longer solutions. Also, if matter is conserved and if the total mass M (of the original universe of radius r_0) is finite, then the density ρ of the infinite universe must be zero. The solution given in Section 3 applies only to a finite, spherical, expanding universe; and *the cosmological hypothesis does not hold*, except as an approximation over a limited region.

This conclusion should not be construed as a reflection on the importance of the work of Milne and McCrea (4). In fact, their discovery of the Newtonian technique in cosmology seems of great value, particularly in view of the unfruitfulness of the relativistic approach.

5. EXPANDING MODEL, MILNE II

Milne I was obtained without any alteration in the Newtonian equations. Now consider a more general treatment, where the gravitational force is represented by $Kr\rho(t)$ on the right side of Eq. 3*a*. If Eq. 5 holds, the differential Eq. 6 still applies, irrespective of K.

As a special case, we now take the *unaccelerated model* used by Milne in nearly all his work (7). Here $K = 0$, so Eq. 3*a* becomes

$$v\frac{\partial v}{\partial r} + \frac{\partial v}{\partial t} = 0, \qquad (3b)$$

with a solution

$$v = \frac{r}{t + a}. \tag{14}$$

Suppose that the universe starts expanding from an original radius r_0 at $t = 0$, and that the maximum velocity is c. Then

$$a = r_0/c, \tag{15}$$

the radius of the universe is

$$r_1 = r_0 + ct, \tag{16}$$

and the velocity is

$$v = \frac{r}{t + r_0/c}. \tag{14a}$$

The total mass is

$$M = \frac{4\pi}{3}\,\rho(t)\cdot r_1^3(t)$$

or

$$\rho = \frac{3M}{4\pi c^3(t + r_0/c)^3}. \tag{17}$$

Characteristics are shown in Figs. 4 and 5. Particularly simple results are obtained by setting $r_0 = 0$, as done by Milne (4).

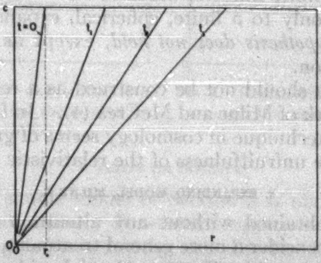


FIG. 5. The velocity is always a linear function of radius. The maximum velocity always occurs at $r = r_1$ and is always c. (Milne II).

Various other values of K may be employed to obtain expanding universes with positive or with negative accelerations. But in all cases, the *universe is finite and the cosmological hypothesis is not satisfied*.

6. THE COSMOLOGICAL CONSTANT

In this family of expanding models, Sections 3 and 5, the gravitational force is $Kr\rho(t)$ and K may be written

$$K = -\frac{4\pi G}{3} + \frac{c^2\lambda}{3\rho}. \tag{18}$$

The quantity λ was originated by Einstein in 1917 and is called the *cosmological constant.* Its purpose is to introduce an *ad hoc* force of repulsion and thereby allow an expanding universe. In Milne I, the universe expands because of the initial velocity of its particles, despite the fact that $\lambda = 0$. In the other models (Section 5), however, $\lambda > 0$. In Milne II, $K \equiv 0$, and thus λ must be adjusted so that for every point and for every instant, the two terms of Eq. 18 exactly cancel.

The idea of the cosmological constant has always been a disturbing one. In fact, the concept seems at least as repugnant and as hard to visualize as the concept of an aether. We shall attempt to show, however, that an associating λ with a modification in Newtonian gravitation, a reasonable picture is obtained.

If $K \equiv 0$, for instance, the Newtonian gravitational force must be cancelled completely over the entire universe. But we know that Newtonian attraction occurs in the solar system and even within galactic clusters, though repulsion may occur between clusters. This inconsistency—the rigid requirement that $K \equiv 0$ everywhere, contrasted with the experimental fact that K is negative for moderate distances—is a result of the usual treatment of the cosmos as a continuum or as a sort of fluid. If the discrete nature of the galaxies is taken into account, the inconsistency disappears.

Our Eq. 1 allows galactic recession by introducing the function $\Phi(\xi)$, which must be negative at small distances and positive at large distances. Data are not yet sufficient to allow an exact specification of $\Phi(\xi)$, though one may assume tentatively that

$$\Phi(\xi) = \frac{\dfrac{\xi}{b} - 1}{\dfrac{\xi}{b\Phi_\infty} + 1}, \tag{19}$$

where b is the distance at which $\Phi = 0$, and Φ_∞ is the value of Φ at $r \to \infty$. This function is indicated in Fig. 6.

If b be taken as 2×10^6 lightyears, the effect of $\Phi(\xi)$ on the behavior of planetary systems, binary stars, and even stellar motions within a galaxy, is negligible. Also, gravitational forces within the local group will still be attractive, tending to consolidate the cluster; but the forces between galactic clusters will be repulsive because of the greater distances. For instance, if $b = 2 \times 10^6$ and $\Phi_\infty = +1$, and we consider the gravitational effect between the local group and the nearby Virgo cluster ($\xi \cong 15 \times 10^6\, ly$), Eq. 19 gives a repulsive force with a numerical value of 77 per cent of the Newtonian value. For more-distant galaxies, the repulsion will approach 100 per cent of the magnitude of the Newtonian force.

If Eq. 1 be applied as if the cosmos were a fluid, no satisfactory model seems possible. But if we realize that galaxies are discrete, with large spaces between clusters, a simple treatment is obtained. Because the distances involved are large compared with b, $\Phi \cong \Phi_\infty$ and the integral of Eq. 3*a* reduces to a summation of terms of the form

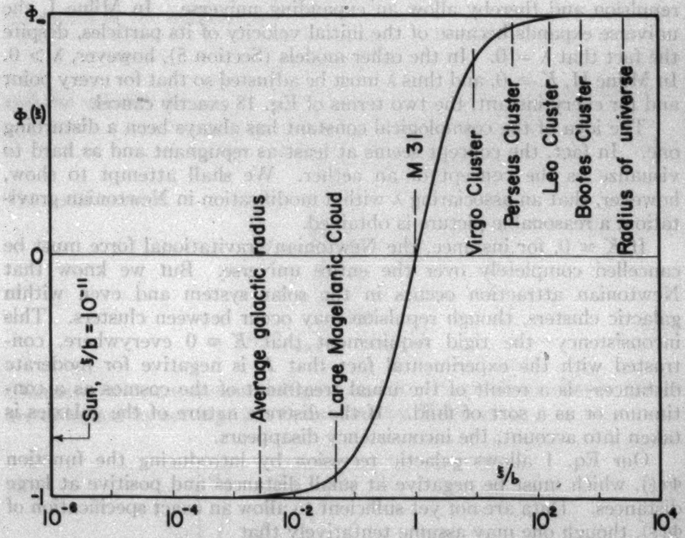


FIG. 6. Proposed gravitational function of $\Phi(\xi)$ for $\Phi_\infty = +1$, giving an attractive force for $\xi < b$ and a repulsive force for $\xi > b$.

$$\mathfrak{F}_2 = +\, \mathbf{a}_\xi \frac{G m_1 m_2}{\xi^2} \Phi_\infty. \tag{1a}$$

Consider a galaxy at point P, distant r from 0. The inverse-square relation of Eq. 1*a* allows the neglection of all galaxies beyond radius r, as in ordinary Newtonian potential theory. Thus the total force at P

is outward, caused by all the matter within a sphere of radius r, and Eq. 1a becomes

$$v\frac{\partial v}{\partial r}+\frac{\partial v}{\partial t}=+\frac{4\pi}{3}G\rho r\Phi_\infty. \tag{3c}$$

The constant K, which was employed in Section 5, is expressible in terms of the arbitrary constant Φ_∞:

$$K=\frac{4\pi}{3}G\Phi_\infty. \tag{20}$$

Also, from Eq. 18,

$$\lambda=\frac{4\pi G\rho}{c^2}[1+\Phi_\infty]. \tag{21}$$

For Milne I,

$$\Phi(\xi)=-1,$$
$$K=-\frac{4\pi G}{3},$$
$$\lambda=0.$$

For Milne II,

$$\Phi(\xi)=0,$$
$$K=0,$$
$$\lambda=\frac{4\pi G\rho}{c^2}.$$

By choosing any other value of Φ_∞, we obtain another model, but all of them are alike in not satisfying the cosmological hypothesis. We have purposely omitted models with continuous creation of matter, though they can also be handled by the Newtonian method.

7. CONCLUSIONS

Forty years of intensive work with general relativity have failed to produce a satisfactory cosmology. This fact raises a suspicion that the cosmological hypothesis is at fault—that a better approach to cosmology is a modification of the Newtonian method aided by all experimental facts now available. Conclusions may be summarized as follows:

1. The cosmological hypothesis should be rejected.
2. Newtonian models do not satisfy the cosmological hypothesis, despite frequent statements to the contrary in the literature.
3. Newtonian models are not equivalent to relativistic models.
4. For very large astronomical distances, Newton's gravitational equation needs revision. A proposed change in the equation allows the usual Newtonian attraction within galactic clusters but provides a possible repulsion between clusters to allow an expanding universe.

REFERENCES

(1) A. EINSTEIN, "Zum kosmologischen Problem der allgemeinen Relativitätstheorie," *Preuss. Akad. d. Wissen.*, Berlin, Sitz., 1931, p. 235.
(2) E. A. MILNE, "Relativity, Gravitation, and World-Structure," Oxford Univ. Press, 1935, p. 24.
(3) H. BONDI, "Cosmology," Cambridge Univ. Press, 1952, p. 67.
(4) E. A. MILNE, "A Newtonian Expanding Universe," *Quar. J. Math.*, Oxford, Vol. 5, p. 64 (1934);
W. H. MCCREA AND E. A. MILNE, "Newtonian Universes and the Curvature of Space," *Quar. J. Math.*, Vol. 5, p. 73 (1934).
(5) W. H. MCCREA, "Newtonian Cosmology," *Nature*, Vol. 175, p. 466 (1955).
(6) D. LAYZER, "On the Significance of Newtonian Cosmology," *Astron. J.*, Vol. 59, p. 268 (1954).
(7) E. A. MILNE, "Kinematic Relativity," Oxford Univ. Press, 1948.

THE FRANKLIN INSTITUTE

MEMBERSHIP

The following elections were approved on June 5, 1958 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Alexander Biddle
C. Glen Bigelow, Jr.
Maurice J. Blocklyn
Allen T. Bonnell
Arthur E. Carlson
John W. Carroll
Joseph A. Connor
Charles S. Crawford
Wilfred W. Delamater
Joseph C. Devine
C. J. Dockhus
William F. Fell
Susie A. C. Field
Albert S. Filemyr
Leonard Finkel
Charles W. Fleming
Joseph J. Flynn
Francis H. Foord
William P. Foster
Alfred K. Frambes
M. Michael Freedland
Joseph A. Ganster, Jr.
John B. Garvin
Charles B. George
Maxwell Gordon
Frank G. Grew
William L. Healy
William A. Kester
Edward R. Kletke
André Leroux
John H. Lux
James J. Lynn
Terrence L. McLaughlin
Irvin Miller
J. Howard Ogle
William W. Omin
William P. Parks
John R. Partin
Alfred E. Pfahler
Jacob Sherman
Joseph E. Sipowicz
John G. Tappert
John J. Tiedeken, Sr.
Robert A. Trenner
Welland J. Welsch
Albert J. Zvarick

EDUCATOR

Raymond D. Fidler
A. Ward France
Mrs. Frank C. Freeman
Otto Paul Fuchs
William A. Opel
Israel Packel
Elias W. Packman

NECROLOGY

H. Carl Albrecht '24
Walter G. Campbell '46
Kenneth S. M. Davidson '53
Christian M. Gottschau '53
Myron J. Hess '55
Joseph W. Little '42
Stirling W. Moorhead '45
Martin William Offinger '54
Charles Penrose '11

COMMITTEE ON SCIENCE AND THE ARTS

(*Abstract of Proceedings of Stated Meeting held Wednesday, June 11, 1958.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 11, 1958.

MR. JOSEPH GRAY JACKSON in the Chair.

The following reports were presented for final action:

No. 3214: The Atomic Clock.

This report recommended the award of a Certificate of Merit to Harold Lyons, of Culver City, California, "In consideration of his work in pioneering the development of clocks of very great precision based on the natural periods of vibration of atoms and employing microwave techniques."

No. 3286: Rosenberg's Cross-Field Dynamo.

This report recommended the award of a Howard N. Potts Medal to Emanuel Rosenberg, of Bogota, Colombia, "For his conception of a fundamentally new principle for deriving electrical current from a rotating machine and for his solution of the many technical problems related to the development of the Cross-Field Generator."

No. 3313: Work of William N. Goodwin, Jr.

This report recommended the award of a Howard N. Potts Medal to William Nelson Goodwin, Jr., of New Hope, Pennsylvania, "In consideration of his many contributions to the field of electrical measuring instruments of which the thermal ammeter, the vacuum tube tester, and the rectifier voltmeter are examples, and in particular for his work on photoelectric exposure meters and establishing a scale for film speeds."

No. 3323: Work of George S. Crampton on the Development of the "Borescope."

This report recommended the award of an Edward Longstreth Medal to George S. Crampton, of Philadelphia, Pennsylvania, "In consideration of his development of the 'Borescope,' a specialized periscope for the internal inspection of turbine shafts, rotors, guns, and various closed vessels, by which flaws may be detected thus providing protection of vital importance from the physical and economic hazards of operational failure."

No. 3343: Clamer Medal.

This report recommended the award of the Francis J. Clamer Medal to Julian Miles Avery, of Framingham, Massachusetts, "In consideration of his discovery and application of the principles of high top-pressure operation of blast furnaces which have very materially increased their production and contributed to their smoother and more economical operation."

No. 3344: Brown Medal.

This report recommended the award of the Frank P. Brown Medal to Charles Milton Spofford, of Brookline, Massachusetts, "For the engineering, aesthetic and educational accomplishments of his life work, exemplified in his prompt recognition, espousal and clarification of valid structural theories; his design of many large and beautiful bridges, and other prominent engineering works; his contributions to engineering education through teaching and administration of an engineering educational department; and his authorship of widely used textbooks."

No. 3346: Henderson Medal.

This report recommended the award of the George R. Henderson Medal to Arthur H. Morey, of Erie, Pennsylvania, "In consideration of his engineering contributions to, and his effective leadership in, the over-all design, development and service testing of the successful 4800 h.p. General Electric Company Gas Turbine-Electric Locomotives which involved the close and skillful coordination of the specialized developments by the Turbine, and other component divisions of the General Electric Company."

D. S. FAHRNEY,
Secretary to Committee

V.2
195

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 583 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, and Fridays; from 2 P.M. until 10 P.M. on Wednesdays; and from 9 A.M. until Noon on Saturdays.

RECENT ADDITIONS

AERONAUTICS

Mises, Richard von and Hohenemser, Kurt. Fluglehre. Ed. 6. 1957.
Stewart, Oliver. First Flights. 1958.

ASTRONOMY

Fraser, Ronald George Juta. Once Round the Sun. 1957.
Moore, Patrick A. Guide to Mars. 1957.

BIBLIOGRAPHY

American Society of Mechanical Engineers. Seventy-Seven Year Index, Technical Papers, 1880–1956. 1957.
National Book League. Science for All; an Annotated Reading List for the Non-Specialist. 1958.

BIOLOGICAL CHEMISTRY

Bredemann, Gustav. Biochemie und Physiologie des Fluors und der Industriellen Fluor-Rauchschäden. Ed. 2. 1956.
Moore, Thomas. Vitamin A. 1957.

BOTANY

Leopold, Aldo Carl. Auxins and Plant Growth. 1955.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Advances in Protein Chemistry. Vol. 12. 1957.
American Chemical Society. Division of Petroleum Chemistry. Techniques of Catalyst Preparation. 1956.
Arima, K. and others. Yeasts. 1957.
Berlow, Evelyn; Barth, Robert H. and Snow, John E. The Pentaerythritols. 1958.
Bikerman, Jacob Joseph. Surface Chemistry. Ed. 2. 1958.
Burriel Martí, Fernando and Ramírez-Muñoz, J. Flame Photometry. 1957.
Harned, Herbert Spencer and Owen, Benton B. The Physical Chemistry of Electrolytic Solutions. Ed. 3. 1958.
Langhammer, Günter. Versuche zur Physikalischer Chemie. 1956.
Puplett, P. A. R. Synthetic Detergents. 1957.
Sarry, Brigitte. Eigenschaften und Bau der Atome. 1956.
Smothers, W. J. and Chiang, Yao. Differential Thermal Analysis. 1958.
Watt, George Willard. Basic Concepts in Chemistry. 1958.

DICTIONARIES

Cassell's German Dictionary, German-English, English-German, Rev. Ed. 1958.

ELECTRICITY AND ELECTRICAL ENGINEERING

Benson, Frank Atkinson. Voltage Stabilized Supplies. 1957.

Crane, Peter Henry Gordon. Switchgear Principles. 1957.
Milnes, A. G. Transductors and Magnetic Amplifiers. 1957.
Müller, Claus. Grundprobleme der Mathematischen Theorie Elektromagnetischer Schwingungen. 1957.
Savant, C. J. Basic Feedback Control System Design. 1958.
Simonyi, K. Theoretische Elektrotechnik. 1956.
Smith, Otto Joseph Mitchell. Feedback Control Systems. 1958.
Spenke, Eberhard. Electronic Semiconductors. 1958.
Summer, W. Photosensitors. 1957.
Tschekanow, W. D. Bergbau-Elektrotechnik. Ed. 3. 1956.
Tubes for Computers. 1956.
Tuttle, David F. Network Synthesis. 1958.
Wilkes, Maurice Vincent; Wheeler, David J. and Gill, Stanley. The Preparation of Programs for an Electronic Digital Computer. Ed. 2. 1957.

ENGINEERING

International Conference on Soil Mechanics and Foundation Engineering. 4th, London, 1957. Proceedings. Vols. 1–2. 1957.

GENERAL

Marcosson, Isaac Frederick. Anaconda. 1957.

HOROLOGY

Bruton, Eric. The True Book About Clocks. 1957.
Robinson, Thomas Rufer. Modern Clocks, their Repair and Maintenance. Ed. 3. 1955.

INDUSTRIAL MANAGEMENT

Production Handbook. Ed. 2. 1958.
Smith, J. Sandford. The Management Approach to Electronic Digital Computers. 1957.

MANUFACTURE

Machu, Willi. Oberflächenvorbehandlung von Eisen-und Nichteisenmetallen. Ed. 2. 1957.

MATHEMATICS

Alger, Philip Langdon. Mathematics for Science and Engineering. 1957.
Anderson, Theodore Wilbur. An Introduction to Multivariate Statistical Analysis. 1958.
Eggelston, Harold Gordon. Problems in Euclidean Space. 1957.
Madelung, Erwin. Die Mathematischen Hilfsmittel des Physikers. Ed. 6. 1957.
Oberhettinger, Fritz. Tabellen zur Fourier Transformation. 1957.
Pfluger, Albert. Theorie der Riemannschen Flächen. 1957.
Prachar, Karl. Primzahlverteilung. 1957.
Priwalow, I. I. Randeigenschaften Analytischer Funktionen. Ed. 2. 1956.
Smart, William Marshall. Combination of Observations. 1958.

MECHANICAL ENGINEERING

Palestrant, Simon S. Practical Pictorial Guide to Mechanisms and Machines. 1956.

METALLURGY

Agte, Curt and Ocetek, Karel. Metallfilter. 1957.
Colombier, Louis. Métallurgie du Fer. Ed. 2. 1957.
Hauffe, K. Oxydation von Metallen und Metallegierungen. 1956.
Rostoker, William. The Metallurgy of Vanadium. 1958.

OPTICS

International Union of Pure and Applied Physics. X-ray Microscopy and Microradiography. 1957.

PHYSICS

Broglie, Louis, Prince de. Physics and Microphysics. 1955.
Buchholz, Herbert. Elektrische und Magnetische Potentialfelder. 1957.
Budó, Ágoston. Theoretische Mechanik. 1956.
Glasstone, Samuel. Sourcebook on Atomic Energy. Ed. 2. 1958.
Landshoff, Rolf Karl Michael. Magnetohydrodynamics. 1957.
Meetham, A. R. Basic Physics. 1957.
Sokolow, A. Quantenelektrodynamik. 1957.
Younger, John Elliott. Advanced Dynamics. 1958.

PLASTICS

Butzko, Robert L. Plastic Sheet Forming. 1958.
Floyd, Don Edgar. Polyamide Resins. 1958.

ROCKETS

Bates, David Robert. Space Research and Exploration. 1957.
Caidin, Martin. Vanguard! 1957.
Davis, Leverett; Follin, James W. and Blitzer, Leon. Exterior Ballistics of Rockets. 1958.

SCIENCE

Conant, James Bryant. Harvard Case Histories in Experimental Science. 1957.

DO YOU KNOW OF THE FRANKLIN INSTITUTE'S NEED FOR GIFTS?

Since 1824 The Franklin Institute, through wise and conservative management, has been partially self-supporting.

As an old institution, established in the public interest, The Franklin Institute has tried to earn most of its cost by diligent and productive work. Like many other institutions of a non-profit nature, The Institute has, in these years of an expanding economy and rising spiral of costs, found it increasingly difficult to perform the broad educational services of which the growing nation and territories are ever in need.

Despite an erroneous public impression gained, perhaps, by the excellence and wide range of services performed, that it is a wealthy, richly endowed organization, The Institute always needs funds.

There is a warm satisfaction in giving financial support to an organization that will use it constructively to help advance and improve our type of civilization.

Your gift or bequest, large or small, will be deeply appreciated. We shall be pleased to give any additional information regarding gifts or memorials.

Where property is transferred, title should be in the name of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts.

Please write to: The Secretary, The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania.

THE FRANKLIN INSTITUTE LABORATORIES

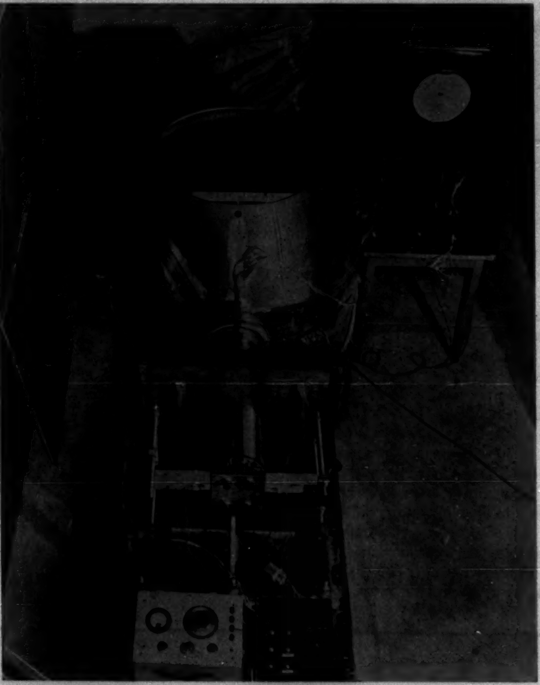
NICOL H. SMITH, DIRECTOR

EXPERIMENTAL STUDY OF STRESSES

BY

ROBERT E. PETERSEN [1]

The model used for these tests is scaled from the 30-in. outlet nozzles in the APDA Fast Breeder Reactor for the Enrico Fermi Power Plant. This reactor is sodium cooled and will be subjected to high temperatures but relatively low pressures. The thin walls allowed by low pressure requirements make it difficult to draw on industrial experience in piping and nozzle design since most previous problems deal with high pressure applications.



Instrumentation and model used in the test.

---

[1] Senior Staff Engineer.

The purpose of this investigation is to determine the relationship between the maximum stresses and applied piping reactions. A theoretical analysis of stresses in nozzles and in the adjacent vessel wall is extremely difficult in view of the complex geometry. In the case of the nozzle considered here the geometry is further complicated by the faired knuckle, and the presence of the conical transition of the vessel wall just above the nozzle. The toroidal fairing was intended to mitigate thermal shock stresses.

Longitudinal moments, circumferential moments, and radial thrusts with respect to the reactor shell were applied to the nozzle. Stresses were determined from strain measurements along the intersections of the vessel wall and a horizontal and vertical plane through the axis of the pipe. Strain measurements were taken in the longitudinal and circumferential directions with respect to the vessel and pipe. These strains were measured on both the outer and inner surfaces. From these data and from given pipe reactions, we can estimate the stresses in the prototype.

Results of these tests indicate that the nozzle stresses are lower than we would have anticipated basing our judgment on Bijlaard's[2] analyses. There are two unique features of the test model which affected the ratios of maximum stress to applied loads; hence the relative roles of the vessel transition knuckle and the toroidal fairing of the nozzle in decreasing the stresses in the shell cannot be compared. It would be desirable to evaluate the toroidal fairing of the nozzle in a vessel without the conical transition.

Acknowledgment

The work described here was done under the sponsorship of Atomic Power Development Associates, Inc.

---

[2] Bijlaard, P.P., "Stresses for Radial Loads and External Circumferential and Longitudinal Moments in Cylindrical Pressure Vessels"; Report to the Pressure Vessel Research Committee, March 1955.

BOOK REVIEWS

FEEDBACK CONTROL SYSTEMS, by Otto J. M. Smith. 694 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $13.50.

The sociological phenomenon of the "vicious circle" has long been recognized as positive feedback. The stabilizing effect of negative feedback has been advocated even longer as illustrated by the quotation "If anyone strikes you on the right cheek, turn the other to him, and if anyone sues for your shirt, give him your coat too." To intelligently apply controls to such feedback systems, there is a need for developing general methods of closed loop analysis and synthesis. This book is written, in the author's words, as "a unified philosophy for the analysis and design of all types of feedback systems." He includes social, economic, government, and biological systems with the more clearly defined physical ones.

The book is designed as a reference and textbook for graduate study, assuming prior courses in basic feedback control and statistical analysis. The four main divisions of the text are: linear analysis, linear synthesis, non-linear analysis and non-linear synthesis and each is treated extensively. The first quarter on linear analysis begins with the conventional treatment of a single feedback loop. It expands into the analysis of multiple loops and a particularly lucid description of power spectra.

The second section, dealing with linear synthesis, opens with the definition of the author's choice of an optimum system, namely, one with minimum error power. The importance of a statistical description of the expected input is stressed. A new function is introduced, called the "realizability operator" which removes, somewhat magically, all unrealizable poles in the optimum design. Coming back down to earth for an interval, the author presents an excellent section on design techniques, indicating an intimate knowledge of the mechanics of synthesis by graphical methods. These techniques are extended to cover the simpler non-linear systems having dead time and distributed parameters.

The third and fourth sections present what is probably the most comprehensive approach to non-linear systems to date, even considering the excellent work of John Truxal. Relay systems with and without multilevel signals are treated at length by the method of two loci in the L-plane. Chapters on unilateral and bilateral non-linearities and discontinuities follow. Examples of these are saturation hysteresis and dead zone as the unilateral type, and backlash and coulomb friction as the bilateral type. Both may lead to discontinuities such as jump phenomena and lock-up oscillation.

Non-linear synthesis is handled primarily by the phase-plane method, graphically and with an eye toward real components. Predictor control for compensating non-linearities is described and a general treatment of carrier systems completes the section. The appendix contains a set of compensating networks with their mathematical and graphical description.

The book is an outstanding contribution to the field, being valuable as a text for advanced graduate study as well as a working reference for the designer of complex systems. It effectively fills out the McGraw-Hill Series in Control Systems Engineering.

CHARLES A. BELSTERLING
The Franklin Institute Laboratories

DISLOCATIONS AND MECHANICAL PROPERTIES OF CRYSTALS, edited by J. C. Fisher, W. G. Johnson, R. Thomson and T. Vreeland. 634 pages, illustrations, 6 ×9 in. New York, John Wiley & Sons, Inc., 1957. Price, $15.00.

This book is the result of a small international conference held Sept. 6–8, 1956 at Lake Placid, New York initiated by N. F. Mott and J. H. Hollomon who recognized the need to review the present state of research both theoretical and experimental on dislocations and their relationship to the mechanical behavior of crystals. It represents the final results of considerable recent work on dislo-

V.2
195

cations in crystals contributed by fifty-four scientists active in this field. Individual papers and summaries of papers which were presented at the conference are included in the book together with edited versions of the discussions which followed each paper. The discussions contributed much in pointing out the need for further work in this field due to a lack of agreement among the participants and complete understanding of many of the associated phenomena.

The book has been logically subdivided into eight chapters which include the following: I, Direct Observations of Dislocations; II, Deformation of Pure Single Crystals; III, Work Hardening and Recovery; IV, Alloy Crystals, Impurities, Yield Point Phenomena; V, Dislocation Damping and Fatigue; VI, Theory of Dislocations; VII, Whiskers and Thin Crystals; and VIII, Radiation Damage.

Considerable work in the observation of dislocations in transparent KCl crystals was contributed by S. Amelinckx. Numerous photographs showing dislocations and the pinning of dislocations into nets are presented and analysed. The results show in a very striking manner the pinning of dislocations by precipitates in addition to the intersection, annihilation and climb of dislocations. Other papers describing techniques for observing dislocations in Silicon, Silver Halides, Lithium Fluoride, Aluminum, Silver and Iron-nickel alloys are given. Direct observations of dislocation markings were generally supported by studies of their movement under applied stresses.

The subject of deformation of face-centered cubic metals by twinning has been a controversial one in recent years and a very interesting paper by T. H. Blewitt, R. R. Coltman and J. K. Redman on the deformation of copper single crystals at low temperatures was presented in which twinning was observed in crystals pulled in tension at 77.3° K. Other work of the deformation of pure single crystals involved the subject of surface sources and their importance in the onset of plastic flow.

A very detailed and comprehensive analysis of the mechanism of glide and work hardening in face-centered cubic and hexagonal close packed metals based upon a dislocation model is presented by Alfred Seeger in which the three stages easy glide, work hardening and recovery are discussed.

The theory of hardening due to the elastic and chemical interaction of solute atoms with dislocations and stacking faults is often used to explain the yield strength of binary alloys. The work of Hideji Suzuki presented in Chapter IV deals with the evaluation of the yield strength under various conditions of elastic and chemical interaction and compares them with experimental observations.

Internal friction phenomena and fatigue have also been analyzed in terms of a dislocation model but due to the complexity of these processes advances in this field have been impeded. Theoretical models capable of describing some aspects of damping phenomena have been proposed. The behavior of metals under reversed stresses is treated by N. F. Mott in which an attempt is made to improve current theory of work hardening under cyclic stress.

Additional important contributions to the theory of dislocations are made by G. Liebfried on the thermal motion of dislocations, by Alfred Seeger on the nonlinearity of the elastic laws in the vicinity of a dislocation, by A. H. Cottrell on the intersection of gliding screw dislocations and by J. C. Fisher on prismatic loops as Frank-Reed sources. Discussions of these papers by other noted authorities in the field proves to be extremely interesting and enlightening.

Radiation damage in metals is the subject of the final chapter in which the study of interstitials and vacancies is discussed. Although far from complete some of the more important aspects of cavity formation and its effect on electrical and mechanical behavior is covered.

Many other papers are presented and discussed in the book which will be of considerable value to many workers active in the field. This volume is not a text book from which to study the complete theory of dislocations. It does, however, represent the accumulation of experimental work, analyses and thinking of numerous scientists presently active in the field of dislocations and will best serve the researcher or scholar already well versed and familiar with the subject of dislocations.

VICTOR DAMIANO
The Franklin Institute Laboratories

THREE DIMENSIONAL DYNAMICS, by C. E. Easthope. 277 pages, diagrams, 5½ × 8½ in. New York, Academic Press, Inc., 1958. Price, $7.80.

Although there are still colleges in this country where a student can graduate in physics without ever coming into serious contact with vector analysis, they are definitely the exception instead of the rule. Even in engineering schools, vector treatment of electromagnetics is common, and that of dynamics becoming so.

A few years ago there were almost no suitable texts which joined vector analysis to technical mechanics. There still are not many, but they are appearing at an increasing rate. Unfortunately, the present work, despite its promising title, will probably find little use in American schools. An intermediate text, it covers material up to Euler's equations but not beyond. The exposition is entirely mathematical, after the British manner. Although the author's language is lucid and relaxed, his excessive formality, highly artificial examples, meager illustrations, and proof-type problems should prove grave handicaps in a U. S. classroom. Perhaps this text would be most useful in teaching vector methods to students already familiar with mechanics.

WILLIAM C. YAGER
The Franklin Institute Laboratories

ELECTRON TUBES AND SEMICONDUCTORS, by Joseph J. DeFrance. 288 pages, illustrations, 6 × 9 in. Englewood Cliffs, N. J., Prentice-Hall, Inc., 1958. Price, $9.00.

This is a very elementary book serving to introduce tubes and semiconductor devices to persons who have little acquaintance with the engineering aspects of radio or electronics. A similar book was published in 1936 by E. D. McArthur, but this earlier work contained more fundamental information relating to physical principles. In that book Richardson's equation was presented and the improvement made upon it by Dushman, so that the student obtained some solid background of electronic principles. He was thereafter able to discuss these effects by name and show some intelligence when, for example, the Schottky effect was mentioned.

Joseph DeFrance's book on the other hand appears to be primarily a collection of tube manual illustrations and characteristic curves. The explanations also bear a resemblance to those presented in the opening pages of the tube manuals or advertising literature. Most technicians are surprisingly familiar with this material and usually possess a large degree of knowledge concerning the practical aspects of the associated circuits.

Since the book is aimed at the "technical institute" or technician level, it would seem better to teach some basic electronic principles, for it is here that the technician's knowledge is most frequently lacking. On the other hand, the kind of information which this author has chosen for his book is usually acquired by technicians from informal sources. The volume might bridge certain gaps in the technician's general knowledge, for it does cover in a comprehensive manner the newer subjects such as transistors and recent advances in photoelectric devices. However, it is difficult to see how such qualitative information could substantially increase one's effectiveness at the working level.

C. W. HARGENS
The Franklin Institute Laboratories

ELECTRICAL DISCHARGES IN GASES, by F. M. Penning. 75 pages, diagrams, 6 × 9 in. New York, The Macmillan Co., 1958. Price, $3.00.

In the introduction, W. de Groot explains that this is a translation of a book published in Dutch in 1955, two years after the author's death. He says further that it offers "a concise synopsis which can profitably be used . . . as a basis for further study. The contents naturally reflect to a great extent the nature and trend of the investigations carried out in the Philips Eindhoven Laboratories, in which investigations Dr. Penning had such a very important share."

This little book (including index and bibliography, it is only 75 pages long) explains, in descriptive terms, the underlying physical mechanisms of different kinds of direct current discharges simply and clearly. Not a comprehensive treatise, it makes no pretense of covering every possible topic, nor does it present any complicated mathematics. Nevertheless, it contains a wealth of detail, and includes equations and numerical data to a sufficient extent to give the reader a feel for the magnitude of the important quantities. Even the reader who has only a very sketchy notion of the nature of electrical discharges will find the explanations easy to follow, conceptually illuminating, and presented with stylistic and pedagogical excellence.

G. P. WACHTELL
The Franklin Institute Laboratories

ADVANCES IN ELECTRONICS AND ELECTRONIC PHYSICS, Vol. IX, edited by L. Marton. 347 pages, illustrations, 6 × 9 in. New York, Academic Press, Inc., 1957. Price, $9.00.

The advent of the International Geophysical Year has sparked research in many fields but in perhaps no one are the advances so significant as in the field of electronics. Not only have advances been achieved in instrumentation application and the use of electronics as a tool, but also as probes in analyzing terrestrial and atmospheric phenomena.

Five contributors cover subjects dealing with the atmosphere, the Aurora Borealis, Negative Ions, Radio Observations of Meteors, Intensity Variations in Cosmic Rays and a review of Radio-Wave Propogation comprise this section. The remaining two sections deal with Electronics in Oceanography and Contributions of Electronics to Seismology and Geomagnetism.

The variety of topics in a single volume makes it difficult to assess this book. This reviewer has had professional interest in the aurora, meteors and seismology. Thus, out of seven sections, only three are of sufficient importance to maintain interest. If the book were intended primarily for electronic engineers and physicists there isn't enough meat in these sections. It is unfortunate that Dr. Marton tried this experiment of using a theme and crossing disciplines instead of restricting the book to a single discipline.

Despite these shortcomings *Advances in Electronics and Electronic Physics* contains some excellent presentations and is a most welcome addition to our libraries.

I. M. LEVITT
Fels Planetarium of The Franklin Institute

ORGANIC ELECTRODE PROCESSES, by Milton J. Allen. 174 pages, diagrams, $5\frac{3}{4} \times 8\frac{3}{4}$ in. New York, Reinhold Publishing Corp. 1958. Price, $6.50.

The reviewer cannot help but compare this book to the section on "Electrolytic Reactions" by Sherlocke Swann, Jr. in "Technique of Organic Chemistry" Volume II, second edition (Interscience, 1956). Dr. Swann's treatment contains an extensive table of electrochemical reactions which makes the data easily accessible to the reader with a minimum of text. He includes a number of illustrative procedures for typical reactions and a brief discussion of electrochemical theory. Dr. Allen's book presents essentially the same information less economically, but does contain more practical information on instrumentation and technique of electrolysis.

M. M. LABES
The Franklin Institute Laboratories

VIBRATION AND IMPACT, by Ralph Burton. 310 pages, diagrams, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1958. Price, $8.50.

In this introductory text, written for advanced undergraduates, the author has intended to convey an understanding of oscillatory phenomena in general, as an introduction to the more specialized subjects of interest to mechanical engineering students. He succeeds in making his presentation attractive through a judicious combination of theory and practical applications.

The history of vibration study, and a number of examples illustrating its importance, are briefly outlined in the first chapter. Succeeding chapters deal with the basic topics of free vibrations of systems having one degree of freedom; typical vibration problems encountered in machinery; damping; and steady forced vibrations. The analysis of periodic non-harmonic forcing functions leads over into a discussion of impact and other forms of transient loading, by way of Fourier series. Nonlinear vibrations are discussed in Chapter 7, and systems with two degrees of freedom are dealt with in Chapter D, which contains a brief but adequate demonstration of the orthogonality of the principal modes.

Chapter 8 presents a useful digression into vibration measuring instruments and the simpler electrical analogs of mechanical systems. A related chapter, No. B, deals briefly with control and feedback systems. The analytical methods employed here are admittedly limited, and do not allow the ease of manipulation permitted by the more standard Laplace transform methods.

Two standard methods of computing natural frequencies of systems with many degrees of freedom are described in Chapter 10. Analyses of vibrations in continuous systems (including gas columns, strings, beams and plates) are outlined. A final chapter discusses the relationships between vibration and fatigue phenomena.

A summary, list of references and problems (with solutions) conclude each chapter. Many problems are worked out in full, in keeping with the pedagogic intent of the book. Much trouble has been taken with the illustrations, although confusion sometimes results from a combination of isometric and diagrammatic views. On the whole, however, this book, on account of its generally clear presentation and interpretation of physical phenomena, can be highly recommended as an introduction to an increasingly important subject.

KLAUS CAPPEL
The Franklin Institute Laboratories

THE PENTAERYTHRITOLS, by Evelyn Berlow, Robert H. Barth and John E. Snow. ACS Monograph 136, 317 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $10.00.

The brief history of pentaerythritol might read, "white crystal—discovered 1882; white powder—multi million dollar industry, 1958." The properties, compounds, and uses of this material are summarized and referenced in this volume.

The wide number of uses for pentaerythritol stems from the high reactivity of its four hydroxyl groups. Thus, many compounds are available for such diverse tasks as filling charges for artillery shells, plasticizers to improve the properties of plastics, preparation of synthetic drying oils for paints and varnishes, and a base material for insecticides. Further uses for fatty acid esters of the material may be in high temperature lubricants for turbojet and turboprop engines.

Discussions of preparations and properties of the compounds of the pentaerythritols are presented according to type. Each chapter contains a catalog of compounds and is well referenced with as many as nine pages of bibliography in the case of nitrates. It is somewhat upsetting, however, to find the uses for pentaerythritol scattered through the chapters. Such format will, of necessity, force the reader who is interested in applications of the material to ferret out the information. It would seem that a separate chapter, inserted possibly after the chapter on preparation of the parent compound, would be desirable.

The Pentaerythritols is not a book for the reader of general chemical interest. It should, however, be recommended for the chemist engaged in work involving the use or manufacture of the material or its compounds, and it will undoubtedly be a valuable addition to the shelves of the technical reference library.

RICHARD HOLLINGER
The Franklin Institute Laboratories

HEAT TRANSFER, Volume II, by Max Jakob with the technical and editorial assistance of Stothe P. Kezios. 652 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1957. Price, $15.00.

The multitude of devoted students, colleagues, and friends of Dr. Max Jakob who mourned his death in January 1955 were saddened additionally by the prospect that the much awaited second volume of Heat Transfer would never be published. The scientific community, therefore, owes a debt of gratitude to Dr. Jakob's wife Anna who had the desire to have the volume made ready for publication and especially to Dr. Kezios whose efforts made that desire a reality. Both volumes stand as a lasting contribution by a kind, humble, and learned man.

Volume II follows the same formal organization presented in Volume I with the sections, chapters, and equations being numbered consecutively through both volumes. The table of contents contains the many subdivisions comprising each chapter so that a specific development within a major topic may be located quickly and easily. Of significant value to the student is the detail paid to the historical aspect of the development of particular topics.

V.2
195

The first two chapters of the book deal with thermal radiation through absorbing and non-absorbing media. In Volume I the laws of thermal radiation and the influence of material properties on radiation were presented without considering the medium between two radiating surfaces; geometrical factors were mentioned but not analyzed. In Volume II the medium on and between two radiating surfaces as well as the geometry of the surfaces is treated in a complete manner. Virtually all possible conditions involving simple configurations are analyzed in a manner sufficiently rigorous to satisfy mathematicians, physicists, and engineers. Certain recent developments, for example, the determination of heat transfer from strongly shocked air are not discussed but undoubtedly will be included in later editions or supplements. Radiation to and from gases is restricted to systems comprising carbon dioxide and/or water vapor.

The major portion of the book has been devoted to selected fields of application. Heat transfer in thermometry, heat exchangers, regenerators, boundary layer heat transfer, and heat transfer in liquid metals are treated in some detail. Of special interest to those working with high fluid velocities are Chapters 38, 39 and 40 which deal with heat transfer through laminar and turbulent boundary layers as well as the fundamentals of transpiration cooling. A thorough discussion of analytical and experimental work up to 1953 is presented.

The last part of the book comprises supplements to various chapters of Volume I including some recent data on physical properties of liquid metals. Additional problems relating to subject materials in Volume I are also included.

An extensive bibliography and author index is presented for the material covered in the book. The researcher's task is made much simpler by virtue of the completeness of the reference list.

This reviewer recommends the book, indeed both volumes, for the engineer or scientist who requires a reference to the fundamental laws and theories of heat transfer along with sufficient applications to indicate the validity and limitations of the theories.

ELLIOTT L. KATZ
The Martin Company

BOOK NOTES

ANALYTICAL MECHANICS, by F. W. Van Name, Jr. 171 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $5.75.

A readable, elementary account of dynamics for physics majors, presupposing no more than general physics and calculus. Designed expressly as a teaching text, it includes only material which can be effectively covered in one semester. Vectors are introduced at the beginning and used throughout.

ATOMIC PHYSICS AND HUMAN KNOWLEDGE, by Niels Bohr. 101 pages, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $3.95.

This is a collection of seven articles by the Nobel prize winner whose earlier work forms the basis of present atomic physics. Four of the articles have been published during the period from 1933 to 1955; the other three are addresses delivered on various occasions but not published previously.

These articles deal with the impact which atomic physics has had on human knowledge, in such fields as biology, anthropology, psychology, natural philosphy and physiology. The volume, representing the attitude of one of this century's greatest scientific minds, will be welcomed as a lucid treatment of a subject which profoundly affects us all.

WHAT'S HAPPENED TO OUR HIGH SCHOOLS?, by John Francis Latimer. 196 pages, 6 × 9 in. Washington, D. C., Public Affairs Press, 1958. Price, $3.25.

Appropriately published at a time when the United States is taking shocked inventory of the deficiencies in its educational system, this book has been several years in preparation.

The author, Assistant Dean of Faculties at George Washington University, has gathered the statistics for his book from many sources, including the U. S. Office of Education and the National Education Association.

The book opens with a history of public education in the United States and proceeds to give data on enrollment and subjects taught for the years 1900, 1910, 1922, 1934 and 1949, showing the extension of educational opportunities and expansion of the curriculum.

In a chapter called "Split-Level Education," the author discusses the division of secondary education into two groups—one for college preparatory students and one for those not planning to attend college. The gradual development and separation of the two curricula are described in detail, with sections devoted to the Dewey philosophy that the aim of education should not be exclusively intellectual, but social as well.

The effect of this split-level curriculum is given in Chapter VI, in which the author traces the decline in enrollment for such subjects as mathematics and the sciences. This chapter also carries the study into the college level, with tables of majors listed for the period 1948–1955, reflecting the trend away from the "classical" subjects.

In the final chapter, the author appraises the situation and offers several suggestions, among them one that the students be grouped according to ability, but that the less gifted child's education should differ only in quantity, not in kind, from that of the gifted child.

This is a solidly documented work, on a basic problem facing us today. It should be of intense interest to teachers, educationists, parents, school board members and faculty members of colleges and universities.

APPLIED MATHEMATICS FOR ENGINEERS AND PHYSICISTS, by Louis A. Pipes. Second edition, 723 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.75.

Substantial additions have been made in this new edition of a well known work. It remains primarily a reference, but is suitable also as a text in the standard omnibus course in applied mathematics. Coverage is strong in most topics, notably matrices, but sketchy on a few. A substantial bibliography is included, partially annotated. There are many problems, but no answers.

MODERN COMPUTING METHODS, by members of the staff of the Imperial College of Science and Technology. 129 pages, diagrams, 6 × 9½ in. New York, Philosophical Library, 1958. Price, $8.75.

This text renders a condensed account of general computing methods with emphasis being placed on practical and simple problems and worked examples. Algebraic equations and matrices, finite difference methods and ordinary and partial differential equations are topics dealt with briefly in the twelve chapters. Included also is an extensive bibliography.

THERMODYNAMICS OF HEAT POWER, by Virgil Moring Faires. Second edition, 432 pages, illustrations, 6 × 9½ in. New York, The Macmillan Co., 1958. Price, $8.00.

This is a second edition, appreciably revised, of the author's *Theory and Practice of Heat Engines*. Intended as primarily a theory book, it nevertheless contains much technological detail, including material on heat exchange in reactors. Following the pattern of Professor Faires' other very teachable texts, there are plenty of excellent illustrations and a wealth of numerical problems, almost all with answers.

TECHNIQUES OF PRESSWORKING SHEET METAL, by Donald F. Eary and Edward A. Reed. 472 pages, diagrams, illustrations, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $12.00.

A reference book dealing thoroughly with the processes of sheet metal work. Based on actual plant experience, this text outlines and defines for clarity the design and construction of dies. The authors analyze vector forces and use strength of materials to determine die component size, with a high degree of detail and an extensive coverage of terminology. A vast number of photographs, line drawings, charts and tables illustrate clearly much of the written text.

PUBLICATIONS RECEIVED

The following books have been received in the JOURNAL office for review. Lack of space prevents publication of more than this listing.

FLUID MECHANICS, by Victor L. Streeter. Second edition, 480 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., 1958. Price, $7.50.

CHEMICAL PUBLICATIONS, THEIR NATURE AND USE, by M. G. Mellon. Third edition, 327 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., 1958. Price, $7.00.

COLLEGE CHEMISTRY, by Paul R. Frey. Second edition, 707 pages, illustrations, diagrams, 6 × 9 in. New Jersey, Prentice-Hall Inc., 1958. Price, $6.95.

THE GOLDEN NUMBER, by M. Borissavlievitch. 91 pages, diagrams, 5½ × 7 in. New York, Philosophical Library, 1958. Price, $4.75.

A SHORT HISTORY OF ANATOMY & PHYSIOLOGY FROM THE GREEKS TO HARVEY, by Charles Singer. Second edition, 209 pages, diagrams, 5⅜ × 8 in. New York, Dover Publications Inc., 1957. Price, $1.75 (paper).

THE STORY OF ARCHAEOLOGY, by Agnes Allen. 245 pages, diagrams, 5½ × 7¾ in. New York, Philosophical Library, 1958. Price, $4.75.

ARTIFICIAL STIMULATION OF RAIN, edited by Helmut Weickmann and Waldo Smith. 427 pages, illustrations, diagrams, 7¼ × 9¾ in. New York, Pergamon Press, 1957. Price, $15.00.

MECHANICAL ENGINEERING LABORATORY, by Charles W. Messersmith, Cecil F. Warner and Robert A. Olsen. Second edition, 179 pages, illustrations, diagrams, 8 × 11 in. New York, John Wiley & Sons, Inc., 1958. Price, $5.50 (paper).

THE ILLUMINATED BOOK, ITS HISTORY AND PRODUCTION, by David Diringer. 524 pages, plates, illustrations, 6 × 9¾ in. New York, Philosophical Library, n.d. Price, $25.00.

THE DIRECTION OF RESEARCH ESTABLISHMENTS, a symposium. v.p., illustrations, 6¼ × 9 in. New York, Philosophical Library, 1957. Price, $12.00.

PHYSICS PROBLEMS AND HOW TO SOLVE THEM, by Clarence E. Bennett. 240 pages, diagrams, 5¼ × 8¼ in. New York, Barnes & Noble, Inc., 1958. Price, $1.75 (paper).

ELEMENTARY PRACTICAL ORGANIC CHEMISTRY, Part 3 Quantitative Organic Analysis, by Arthur I. Vogel. Pages 645 to 840, diagrams, 5½ × 8½ in. New York, Longmans, Green & Co., Inc., 1958. Price, $4.50.

CLIMATOLOGICAL SUMMARY MOULD BAY, N. W. T., CANADA. 92 pages, diagrams, 8 × 11 in. Canada, Department of Transport, Meteorological Division, 1957. No price, (paper).

PHILOSOPHY OF MATHEMATICS AND NATURAL SCIENCE, by Hermann Weyl. Third printing. 311 pages, diagrams, 6 × 9 in. Princeton, Princeton University Press, 1958. Price, $1.95 (paper).

CURRENT TOPICS

Rockets Probe Upper Atmosphere for IGY.—A highly successful season of rocket probings of the upper atmosphere was completed in April at Fort Churchill, Manitoba, Canada, as part of the US International Geophysical Year program. Reports reaching the National Academy of Sciences' IGY Committee in Washington indicate that 22 of the 23 firings in 1958 produced the desired scientific data. Since IGY began in July 1957, 41 rockets have been launched at Fort Churchill, the most extensive rocket sounding program ever conducted in the higher latitudes. Firings there were resumed in mid-June, with at least 32 more scheduled before IGY ends.

The rocket firings explored the area between 46 and 160 miles above the earth's surface, a portion of the atmosphere which is out of reach for balloons and, for the most part, below the level of satellite orbits. Eleven types of experiments were conducted in this ionosphere layer, which, in addition to being the subject of basic scientific research, is of immediate practical interest because of its effect on long distance radio communications. Included were the first successful firings into visible auroras.

Data now being processed are expected to throw new light on such matters as the altitude of the electric currents responsible for disturbances in the earth's magnetic field during an aurora and on the nature, extent, and location of the electrons and ions at high altitudes, which are governing factors in radio transmission.

Ingenious techniques were used to measure the vertical distribution of pressure, temperature and density of this segment of the atmosphere. Paradoxially, these techniques were made necessary by the rocket itself. Although the rocket is the only means of attaining the right heights, its great speed does not allow an ordinary thermometer probe to come into equilibrium with the atmosphere. It therefore becomes necessary to calculate temperature indirectly from pressure, density, or velocity of sound.

In the sound-velocity method, 18 explosive grenades are ejected from the rocket at timed intervals. On the ground, an array of sensitive microphones is spaced 1000 yards apart in the form of a cross. Each microphone "hears" the burst at a slightly different time. By noting the time taken for the sound to travel from rocket to ground, average temperature of the air can be computed, and the difference in arrival time at the various microphones yields a measure of average upper level wind velocity and direction.

The Fort Churchill operation is the result of the combined efforts of many agencies. The facility itself was established jointly by the Canadian and US National Committees for the IGY and the defense establishments of both countries. The U. S. Army is responsible for the complicated ground support phase—firing of just two rockets involved 4,408 man-hours of overtime in activities ranging from road blocks for safety to actual launchings, for which the Navy provides firing crews.

Scientific participants from the U. S. include the Air Force Cambridge Research Center, Ballistics Research Laboratory of Aberdeen, Md., Signal Engineering Laboratory of Fort Monmouth, N. J., State University of Iowa, U. S. Naval Research Laboratory, which with assistance of personnel from New Mexico College of Agriculture and Mechanic Arts operates an ionosphere station and telemetering trailers, the University of Michigan, and the University of Utah. Canadian organizations make auroral observations, magnetic recordings and ionosphere soundings.

Rocket vehicles used or planned are the Loki Phase II-Dart, the Nike-Cajun and Nike-Asp, and the Aerobee and Aerobee-Hi. The Aerobees are the largest, and carry 150 pounds of scientific instrumentation.

Other phases of the US-IGY rocket program are carried on at White Sands and Alamogordo, N. Mex., Pt. Mugu, Calif., and from shipboard in the Arctic, equatorial regions and the Antarctic, where the rockets are launched from balloons.

Electronic Highway of Tomorrow in Operation.—A segment of the "electronic highway of tomorrow" went into operation in May at the David Sarnoff Research Center of the Radio Corporation of America at Princeton, N. J. Cars entering the grounds of the research center, headquarters of RCA Laboratories, passed over electronic loops buried in the entrance road. Transistorized detector units connected to the loops counted each of the passing cars, measured the speed of each, and flashed a polite warning saying "Slower Please" to every driver exceeding the posted speed limit.

The new equipment is a transistorized variation of the comprehensive electronic vehicle control system developed earlier by RCA and demonstrated last year at Lincoln, Nebraska, in cooperation with the Nebraska Department of Roads. At Lincoln, the system showed its ability to guide cars automatically along the highway and to provide electrical signals for the automatic control of braking and steering.

The new RCA Laboratories installation is designed to demonstrate further immediate applications of the system in specific traffic control jobs. Dr. Vladimir K. Zworykin, Honorary Vice-President of RCA, directed development of the entire vehicle control system, in cooperation with an RCA Laboratories research team including L. E. Flory, W. S. Pike, and G. W. Gray.

The distinction of receiving the first speed warning fell to Mr. Flory, who purposely drove past the 20-miles-per-hour speed limit sign at the research center entrance at a smart 30-miles-per-hour clip. As the car passed over the two wire loops buried in the pavement, four things happened simultaneously:

1. The transistorized detector units connected with the buried loops detected the passage of the car and measured its speed.
2. Since the speed was greater than the specified limit, the circuits automatically illuminated the warning sign at the roadside some twenty yards ahead of the car.
3. A simple counter connected to the detector units recorded the passage of the car.
4. A second counter automatically recorded the passage of a car in excess of the speed limit.

"The idea of such an installation on the highway would be to study driver habits and to determine whether or not a posted speed limit is realistic," said Dr. Zworykin. "By installing such a system at the entrance to the David Sarnoff Research Center, we will have an opportunity to study the functioning of the equipment over extended periods and under all conditions of weather."

Ultimately, according to Dr. Zworykin, another loop can be installed farther along the road to determine how many drivers slow down after receiving the warning from the automatic sign, "thereby giving us another indication of driver reactions."

Dr. Zworykin emphasized that the newly installed units are components that would be standard in the comprehensive electronic control system designed for the ultimate automatic control of vehicles on the highway. Like the demonstration equipment at the laboratory entrance, the larger system consists basically of wire loops buried in the pavement along a highway, with detector units associated with each loop to pick up the signals generated by passage of cars over the loop. In the complete system, these signals are fed back through antennas under the road to activate automatic controls in following cars, or they are used to operate various roadside warning devices.

"In the complete system," said Dr. Zworykin, "any segment at any point on the highway can be used to perform the counting and speed measurement functions demonstrated in this laboratory installation.

"Various radar and mechanical techniques are now used for this purpose, but these existing devices require special installation wherever they are to be used, and they bear no relation to the complete system in the highway itself. By using elements of the complete RCA system for these specific and limited functions, a variety of installations can be made in the near future with the assurance that they will function also as integral parts of the more extensive highway installations of tomorrow."

Mr. Flory, as a member of the research team responsible for development of the electronic highway control system, gave this brief description of the way in which the newly installed RCA Laboratories equipment operates:

The detector loops in the roadway itself consist of rectangles of wire 6 × 20 ft. in outline. In operation, a voltage from a high-frequency power line is applied to each of the loops. A vehicle passing over the loop causes a variation in the current passing through the wire. This variation is immediately detected by a small transistorized detector circuit placed at the side of the road. The detector circuit then produces an output signal which is used to perform the desired action. In the RCA Laboratories installation, the signals from the detector units activate the counters. In addition, the time interval between the variations in the two loops is measured to determine the speed of the vehicle, and the illuminated sign is turned on if the speed exceeds a predetermined limit.

Consumable Weld Insert for Thick Pressure Vessel Wall.—A consumable weld insert, developed originally for butt welding pipe, has been used successfully by Newport News Shipbuilding & Drydock Co., Newport News, Va., to join the hemispherical halves of a nuclear reactor pressure vessel.

The pressure vessel presented a difficult welding problem in that all welding had to be done from the outside while at the same time the inside of the weld had to be smooth, uniform and crevice free. The 60-in. I.D. pressure vessel is a sphere of 4-in. mild steel internally clad with an 0.4-in. layer of type 347 stainless steel. In operation, the vessel handles a highly corrosive process fluid of heavy water under 2000 psi. pressure.

Although the use of the weld insert requires more careful preparation of the joint than standard welding practice, it assures a high quality weld from one side. Leaving an 0.050-in. land, at the inner surface a $\frac{3}{16}$-in. radius was machined and the joint face turned at 20 deg. for a little over one inch. The remainder of the surface was cut at a 10-deg. taper.

After visual inspection of the joint, the type 347 stainless steel EB insert was tack welded around the 0.050-in. land of one of the hemispheres. This insert, manufactured by Arcos Corp., Philadelphia, is in the form of wire with a special cross sectional shape of a mushroom. The head of this mushroom shaped wire was toward the inside of the sphere.

The two pressure vessel halves were placed in a holding fixture that carefully located the hemispheres to permit the tack welding of the insert to the other side of the joint. The homogeneous reactor sphere was purged and filled with an inert gas to protect the inner surface against oxidation during the root pass and the laying of the next two beads.

All welding was done from the top by putting the holding fixture in the vertical position. Fusing of the weld insert by inert gas shielded tungsten arc from one side only proceeded in 12-in. segments, approximately one eighth of the circumference. One segment was fused and the second bead was overlayed before going to the next segment. The third bead was run over both sections before the fixture was rotated for welding of the two diametrically opposite segments.

After completion of the root pass and the two subsequent passes, the closure was subjected to radiographic inspection with no defects of any sort permitted. After the inspection proved satisfactory, subsequent passes were made with type 347 electrodes to join the stainless steel to within $\frac{1}{8}$-in. of the interface of the stainless steel cladding and the mild steel where type 308L was used for five passes. Armco iron was used for 12 passes and the remainder of the joint was completed with E-7016 electrodes.

Key to the entire weld, performed from one side, is the root pass made with the EB weld insert. This technique produced a smooth, defect-free base obtainable by no other process.

Aluminized Suit to Protect Fire-Fighters.—An expendable aluminized suit for the protection of fire fighters and a unique emergency water supply system were used successfully in large scale fire tests conducted recently in northern California forests by the U. S. Army Research and Development Laboratories, Fort Belvoir, Va.

Preliminary reports on the tests, held to evaluate fire fighting equipment for troop use, indicate that men wearing the suit over duty uniforms were able to stand within two feet of a forest fire for two to three minutes without discomfort, although the heat was so intense that exposed portions of helmet liners were blistered.

Designed by the Engineer Laboratories and fabricated by the Quartermaster Corps, the expendable suit is made of flame-retardant treated aluminized kraft paper. It consists of a parka with hood, a face mask, a pair of leg sleeves, and a pair of mittens.

Other results of the test conducted in cooperation with troops from Fort Lewis, Washington, showed the feasibility of an emergency water supply system, also designed by the Laboratories' Fire Fighting Branch. This consists of lightweight piping and hose, and an electrically-driven submersible pump.

Soldiers inexperienced in fire-fighting were able to couple 1000 ft. of pipe in three and a quarter minutes and the lightweight hose was laid successfully by a helicopter flying at tree-top level.

Helium Wind Tunnel.—Helium, the gas that fills slow-moving blimps, is being used in studies of problems associated with the ultra-high-speed flight of satellites, missiles, and meteorites at the General Electric Research Laboratory. Dr. Guy Suits, GE vice president and director of research, has disclosed that the Research Laboratory is now operating what is believed to be the largest and fastest helium wind tunnel in the world.

The new facility, in which helium serves as a "stand-in" for air, has produced the highest Mach numbers ever attained in a wind tunnel, reaching velocities in helium corresponding to Mach number 28 (28 times the speed of sound), Suits revealed. The attainment of these record-breaking Mach numbers, he added, represents "progress toward better understanding of the problems of high-speed flight."

Speeds corresponding to a Mach number of approximately 25 in air were needed to launch the first U. S. satellite, "Explorer I." A Mach number of 35, based on sea level sound velocity, would be necessary for an object to escape the earth's gravitational field. Mach numbers in this region may be reached in future experiments.

The studies at the GE Research Laboratory, partly sponsored by the U. S. Air Force, make use of a metal tube in which helium flows past a model representing, for example, a missile or its nose cone. In most conventional wind tunnels, huge compressors send a stream of air past the model being studied. In this helium tunnel, the compressed gas is stored in tanks and then allowed to escape through a nozzle into the test section, in which a near-vacuum is maintained. As the expanding gas rushes down the tube past the model, very high velocities, together with very low temperatures and pressures, are produced. Static temperatures only two or three degrees above absolute zero have been obtained at the Research Laboratory during runs lasting from seconds to minutes.

"The growing interest of the General Electric Company in very high speed flight has resulted in a pressing need for a greater understanding of the flow of air above Mach number 20," according to Robert H. Johnson, mechanical engineer conducting the helium tunnel experiments in the laboratory's recently completed $1.2 million Gas Dynamics Facility.

"This interest in the mechanics of fluids is not, however, a recent development for General Electric," Johnson added. "It began with the cut-and-try methods of the early days of the steam turbine and the oscillating wall fan. Today this empirical approach has matured into sophisticated theories that make possible continued improvements in the design of steam and gas turbines, as well as the acquisition of new knowledge in the field of very-high-speed flight."

Helium is used in the tunnel in order to avoid some of the complexities that face the researcher studying high-speed flight in air. At extremely high speeds, air breaks down into an assortment of elements, compounds, and ionized particles that bears almost no resemblance to the air we breathe. Helium, however, the stablest substance in the universe, is not subject to the great changes that occur in air's properties and composition. In addition, helium has an extremely low liquefaction temperature, which ensures that it will not liquefy (as air would) as it passes through the tunnel passages and through the shock waves that occur in front of the model.

As a result of helium's more tractable behavior, it is possible, by analogy, to obtain information on the dynamic and viscous behavior of air at very high Mach numbers. This information, together with data from complementary research tools (such as shock tubes), contributes to a better understanding of the complex actual behavior of air under similar conditions.

Test sections with diameters of either 3.6 or 6 in. can be used in the tunnel, depending upon the Mach number desired. Models up to one inch in diameter can be studied, with data taken by means of schlieren photographs and manometer board readings.

Circulation of Atlantic Studied.—Scientists at the Woods Hole (Mass.) Oceanographic Institution are analyzing extensive data on deep currents of the Caribbean Sea and the Atlantic Ocean brought back from a two months' cruise by the research vessel *Crawford*.

The information collected on the *Crawford*, when considered with other Atlantic IGY data, will shed important light on the circulation of the Atlantic Ocean and particularly on the "age" of deep Caribbean water, according to Dr. Columbus O'D Iselin, Woods Hole director. The Woods Hole effort is part of the National Academy of Sciences' International Geophysical Year Committee program to enlarge man's knowledge of the earth.

The six-man scientific party on board the ship, headed by oceanographer William G. Metcalf, obtained more than 2000 deep temperature measurements and an equal number of water samples which were analyzed for salinity, oxygen content and other chemical constituents. The samples were obtained from the surface down to depths as great as 23,000 feet while the ship was stopped. In addition, some 357 temperature measurements from the surface to a depth of 900 feet were obtained while the ship was under way.

Detailed examination of the observations, and comparison with similar data obtained 30 years ago by the Institution's R. V. *Atlantis*, will help to reveal how old the water is in the enclosed Caribbean Basin. Important are the depths of the "sills" in the passages between Caribbean islands, through which deep Atlantic water cannot enter under present-day climatic conditions. Sills are submarine ridges connecting two land masses; sill depth is the distance from the surface of the water to the top of the ridge.

Using the recently-developed Woods Hole precision echosounder recorder, the *Crawford* determined that the Windward Passage between Cuba and Haiti has a sill depth of 5,046 feet. This passage and the Anegada Passage in the Virgin Islands are the deepest sills leading to the Caribbean and prevent exchange between the coldest Atlantic water and the deep Caribbean water.

The echosounder was constantly in use and obtained accurate bottom profiles along the entire track of the *Crawford*, ranging from Bermuda to the Virgin Islands to Venezuela, from Colombia to Haiti, Jamaica to Central America, back to Cuba and thence to Panama.

To date in the Woods Hole-IGY Deep Current Program, more than 7000 deep temperature and water samples have been taken as well as some 3760 shallow temperatures (down to 900 feet) and bottom profiles along all tracks of the ships.

Through the combined efforts of the (British) National Institute of Oceanography and the Woods Hole Oceanographic Institution, a countercurrent under the Gulf Stream was charted last year. Eight crossings of the North and Central Atlantic Ocean, with some 140 "deep stations" completed en route, also were made by ships of the two institutions. The data from these cruises will shortly be forwarded to the IGY Data Centers.

Helium Separation and Purification.—A diffusion technique which promises to facilitate greatly the large-scale separation of helium from gaseous mixtures such as natural gas has been developed by K. B. McAfee of Bell Telephone Laboratories. Helium with no detectable impurities may also be obtained by using this technique as a purification process.

The demand for helium in the U. S. has risen much faster than the ability to produce it. Our only sources of supply are a few natural gas wells, where helium exists at concentrations ranging from less than 1 to 6 or 7 per cent. Most of this helium at present is wasted when the natural gas is burned for fuel. Any process for readily recovering helium would be of tremendous commercial value.

In the newly developed diffusion process, helium is separated from a gaseous mixture, such as natural gas, by passing the mixture over the surface of glass which has a high permeability to helium and low permeability to other gases. Silica glass has been found to be particularly advantageous in this respect, having a permeability to helium as much as one thousand times greater than for hydrogen, the next most diffusible material. The permeability is high enough to permit relatively large quantities of helium to diffuse through the glass under proper conditions.

To obtain appreciable quantities of helium, a large surface of glass must be exposed to the mixture, the glass must be very thin, and a high pressure differential should be maintained between the two sides of the glass. An excellent configuration for providing such conditions consists of a bundle of fine capillary glass tubing so arranged that the gas mixture flows around the outside of the tubing under high pressure and the helium is recovered from the inside of the capillaries. Appropriate seals have been developed to facilitate such an arrangement.

Silica or pyrex tubing can be drawn having an external diameter of two mils and a wall thickness of two-tenths of a mil. Such tubing can withstand a compressive stress in excess of one thousand atmospheres, and is ideally suited for the separation or purification of helium. Diffusion through the glass increases rapidly with temperature, and the tubing will withstand temperatures of 400° C. and higher over long periods of time without deteriorating.

Tests on an experimental thin wall capillary diffusion cell made up of a bundle of capillaries indicate that a similar cell containing enough capillaries to occupy about two cubic yards would pass nearly 1000 cubic feet per day of helium at room temperature with a pressure differential of 1000 atmospheres, assuming a concentration of 1 per cent helium in the gas mixture can be maintained. Increasing the temperature to 400° C. would permit the recovery of 100,000 cu. ft. of helium per day. A cell of this type will not deteriorate with time, and might be placed directly in a gas pipeline. Thus it appears that this process may have a truly large scale potential.

Exceptionally pure helium results from a single diffusion step, even though the initial mixture contains as much as 90 per cent hydrogen and only 10 per cent helium. A careful analysis indicates that there is less than 0.0009 per cent hydrogen in the helium purified by such a step. Helium purified by diffusion through glass by this process is among the purest gases which it is possible to produce.

Results already obtained indicate that this process can be adapted to large scale commercial purification of helium, and to certain high volume commercial separations involving this gas.

International Lighting Vocabulary.—A new international vocabulary of lighting terms, the culmination of 20 years of study by a working party of the Commission Internationale de l'Eclairage (C.I.E.), is now ready for distribution through the organization's United States National Committee. Printed in three languages—French, English, and German—the vocabulary was created for the express purpose of facilitating communication among scientists the world over who are working in the field of illumination.

The publication, entitled "International Lighting Vocabulary of the International Commission on Illumination, Volume 1 (2nd Edition)," contains 530 terms, with definitions, as well as numerous symbols and formulae. Its principal sections cover Radiation, Photometry, Colorimetry, Eye and Vision, Production of Light, Lamps, Auxiliary Apparatus, Lighting Techniques, Lighting Fittings, and Light Signals.

Volume 2 of the publication will be published in 1959. It will contain the same terms, without definitions, and will be printed in ten languages. These are: French, English, German, Danish, Dutch, Italian, Polish, Russian, Spanish, and Swedish. The material will be arranged systematically, with ten alphabetical indexes.

The United States National Committee has placed a prepublication order for the first volume and is offering it at a reduced price. A limited number of copies are now available at $2.50. It is expected that the price will be raised when the initial supply is exhausted.

Orders for the publication with remittances payable to "U. S. National Committee, C.I.E." should be sent to Mr. T. D. Wakefield, Treasurer, U.S.N.C., Vermilion, Ohio.

Laboratory Automation System.—Complete automation in the chemical laboratory will be introduced in the United States this year. It will be made possible by a system of instruments that will perform the most exacting scientific tests, record their findings, and take proper corrective action when necessary, all without human help. The system will be offered this fall by Chicago Apparatus Company.

Called the Analmatic, the instruments are made by Baird & Tatlock Ltd., Great Britain, and are extensions of a basic system the company designed and built for the British Atomic Energy Authority. The system completely frees the scientist from routine analytical work.

The Analmatic will be displayed for the first time in this country at the National Chemical Exposition by the Chicago section of ACS Sept. 9–12 in Chicago's International Amphitheatre. Chicago Apparatus has been named exclusive United States and Canadian distributor for the Analmatic.

The system is far more comprehensive automation than anything that has been tried in American laboratories. The term "automation" very often is used too loosely to describe single instruments that give a quick reading or short cut the analysis procedure, once the scientist has prepared the testing sample.

Here are the operations performed by the Analmatic instrument system without human help:

1. Collects a succession of exactly measured samples of the material to be analyzed.
2. Mixes the successive samples with metered amounts of reagents, as necessary. For example, analyzing uranium concentrates involves the addition of ammonium thiocyanate.
3. Heats or agitates the solution as required.
4. Performs the necessary test by color measurement, by titration to pH, by potentiometric or "dead stop" end points, by photometric measurement, etc.
5. Records the test results as graph or dot diagrams, or in digital form.
6. Thoroughly washes all the pipettes and other vessels used to hold liquids, insuring that there is no contamination that will affect subsequent tests.
7. If the analysis shows that the test material deviates from a given critical value or range, a visual or aural alarm is given. If desired, the alarm can be replaced by a control system that will adjust the production process automatically, compensating for the previous error.

The instrument system, not an assembly line item, is built with typical British craftsmanship. Price of the instruments will depend on individual requirements, since each system is engineered for the user. Probable price range will be from $15,000 up to $50,000. The reason the range may be so wide is that the system will vary from the relatively simple to the very complicated. In a given case, Baird & Tatlock can engineer a system capable of handling several hundred daily determinations of a complicated chemical assay. In other instances, a relatively simple and less expensive Analmatic is now used by several English firms for the rapid and accurate determination of moisture by the Karl Fischer reagent test.

The first analyzer was put in operation by the British Atomic Energy Authority in South Africa to check the waste flow in a uranium purifying process, guarding against too much uranium being "washed down the drain." In this application, the Analmatic does the work of two skilled analysts who were taking samples at 15-min. intervals. For 24-hr. operation, the manpower saving is tripled. Other current users in England include a manufacturer of penicillin and other pharmaceuticals, the largest chemical firm in the country, and a petroleum products plant.

According to Chicago Apparatus, the following industries in this country are the most likely to have automatic laboratory applications: pharmaceuticals, petroleum processing, many different kinds of chemical production, paper manufacturing, food processing, plastics production.

Analmatic equipment is at present available only for clear liquids. Baird & Tatlock has designed apparatus for external filtration of samples and reagents entering the analyzer, but has not worked on automation techniques for testing slurries, suspensions, or solids.

As American agent, Chicago Apparatus will provide technical assistance for all installations. Actual engineering systems for the first few months will be performed by Baird & Tatlock personnel who will come to this country in August.